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BRIDGES

BY

PROF. FLEEMING JENKIN

BRIDGES

AN ELEMENTARY TREATISE ON
THEIR CONSTRUCTION AND HISTORY

BY

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Reprinted from the Encyclopædia Britannica

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EDINBURGH: ADAM AND CHARLES BLACK
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186. h. 93.

BRIDGES:

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PREFACE.

A few words of explanation are required as to the reason for republishing the present short article. When the writer was asked to undertake the subject of which it treats for the "Encyclopædia Britannica," he was in a great measure induced to do so by the thought that he could so arrange the matter as to produce a short treatise which might serve as a text-book for one portion of the course of lectures delivered by him in the University of Edinburgh. The publishers consented to the immediate republication of the article with this object. There are many existing works on Bridges of far greater value than the present little book; indeed, this branch of engineering is exceptionally well illustrated both by papers on special bridges, such as are found in the minutes of the Proceedings of the Institution of Civil Engineers, and by large and well-illustrated works, such as "Weale's Papers." Moreover, the mathematical examination of the principles by which the form and dimensions of the parts are to be determined has been carried out with great accuracy by a variety of authors. The present treatise does not aim at adding much, if anything, to the existing knowledge either of practice or theory, but the very completeness with which the subject is understood led him to include it as one part of his course of lectures, with the especial object of showing how the theory of the strength of materials was applied in practice. The problems by which the stress on the component parts of the superstructure of a bridge is calculated are not very complicated, and practice has shown very thoroughly the limits within which the results of the calculations may be trusted. The subject, moreover, affords excellent examples of the numerous practical considerations which must guide an engineer in designing any important structure, and also of the rough and ready methods by which experienced men arrive at a very fair knowledge of the probable weight, cost, and general character of a structure without entering into minute particulars or details of the design. The present treatise begins, therefore, with an elementary essay on the strength of materials, and proceeds to show how the principles laid down are applied to the design of the three great types of the superstructure of bridges, namely, girders, arches, and suspension bridges. The use of reciprocal figures in calculating the stresses in frames is explained at some length. A brief sketch is then given of the several methods of founding piers under water, and of the general considerations which should guide the designer in selecting one or another type for a given locality. A slight historical sketch concludes the article. Certainly each part has been both more fully and more ably treated of in other works, but it is hoped that students will find collected within a moderate compass all that is necessary for an elementary study of the design of a bridge; and when the student has mastered this portion of his professional study, he will not find it difficult to extend the same principles to the design of a roof crane or structures in general.

The elementary essay on the strength of materials makes no mention of the complete mathematical theory of the subject, but takes the strictly elementary view, according to which, for instance, a rod under tension is treated as a bundle of parallel fibres, all equally strained, and capable of yielding only in one direction. The fact that this view of the subject is incomplete, and indeed inexact, is well known to physicists, and is forcing itself on the attention of experimental engineers, as may be seen in the works of Mr Kirkaldy, who insists on the importance of the contraction which occurs in the section of the rod while it is being extended. The elementary assumption, incorrect though it be, is the only one in use among engineers, and within certain limits it gives results sufficiently accurate for practical application; but this word of warning that it is not complete may probably be of use.

The student will find that there is not much room for further invention in connection with the design of the superstructure of a bridge. It is curious to observe the essential unity of arch girder and suspension bridge, as illustrated by designs which pass by imperceptible gradations from one type to the other. There is hardly a missing link in the chain. What can be done and what cannot be done with existing materials may almost be said to have been determined.

One step lately made has been the improved design of metal arches, and the writer believes it is possible in this direction to surpass even the great St Louis bridge recently erected over the Mississipi, but the chief advance of late years has been in the methods of founding under water. In this branch of engineering the improvements have been very great, and probably these have not reached their limit. In one direction it may be said that everything remains to be done. The genius has still to come who shall teach us how our metal structures may be made beautiful.

BRIDGES

§ 1. *Definitions and General Considerations.*—Bridges are structures designed to carry roads across streams, gullies, or roads. A viaduct may be distinguished from a bridge, inasmuch as the object of the former is to carry a road at a considerable elevation above the surrounding country, by means of structures, similar indeed to bridges, but in which the object of the open spans is to save expense rather than to cross some obstacle which could not be passed by a level road or embankment. The aqueduct is a structure similar to the viaduct, but employed to convey or support water. A culvert may be distinguished from a bridge as an opening, the primary object of which is to let water flow past a road or other obstacle, the object being similar to that of a large drain. A large culvert might be called a small bridge, and a bridge having long approaches with many spans might be called a viaduct. The present article will treat only of Bridges.

Every bridge may be divided into two parts, the *substructure* and the *superstructure.* The substructure of a bridge consists of foundations, abutments, and piers. The end supports of the bridge are the abutments, and the intermediate supports are called piers. Piers and abutments rest on foundations in the ground. A bridge of one span has no piers. The superstructure of a bridge consists of the roadway and the beam, arch, or chain used to carry the roadway from support to support.

The dimensions and design of a bridge depend on the nature of the obstacle to be crossed and on the traffic to which the road over the bridge is subject. The engineer is usually bound to design the cheapest structure which will perform the required duties; he has, therefore, in each case to consider whether a small number of large spans or a large number of small spans will be cheaper. Large spans will be desirable where foundations cannot be easily obtained, or where the height of the structure is great. The engineer has also to determine whether, considering the prices of materials, labour, and transport, one or other form of superstructure is to be preferred. The traffic to be accommodated will determine the width of the bridge and the load which the superstructure must bear. In many cases it will also be the duty of the designer to endeavour to combine beauty with utility. Beauty does not require ornament or expense, but demands, what may be more difficult to supply, correct taste in the designer.

In Great Britain law prescribes the following minimum dimensions for the over and under bridges of railways. (An *over bridge* is one in which the road goes over the railway; an *under bridge* one in which the road goes under the railway. *Over bridges.*—Width: turnpike road, 35 feet; other public carriage road, 25 feet; private road, 12 feet. Span over two lines (narrow gauge), generally about 26 feet; head room, 14 feet 6 inches above outer rail. *Under bridges.*—Spans: turnpike road, 35 feet; other public carriage road, 25 feet; private road, 12 feet. Head-room: turnpike road, 12 feet at springing of arch, and 16 feet throughout a breadth of 12 feet in the middle; for public road, 12 feet, 15 feet, and 10 feet in the same places; private road, 14 feet for 9 feet in the middle; for exceptions the Acts must be consulted. In designing a bridge to cross a stream care must be taken to insure that the openings are suitable for the maximum floods.

The *load* which the superstructure of a bridge has to carry in addition to its own mass may be estimated as follows:—

1. For a *public road;* one hundredweight per square foot will represent the weight of a very dense crowd. This is greater than any load which ordinary carts or vans will bring on the bridge, but of late years traction engines and road rollers have been introduced, and a weight of perhaps 10 tons on each wheel on one line across the bridge ought in future designs to be provided for. The bridge must be strong enough to bear this maximum weight applied at any point, and also to bear all possible distributions of the crowd. A bridge might be fit to carry the crowd uniformly distributed over its surface, and yet fail when the crowd covered one-half of its length or width.

2. For a *railway;* the maximum passing load on each line of rails may be taken as the weight of a train composed exclusively of locomotives. The bridge must be fit to bear this load distributed in all possible ways along the line. For spans above 60 feet on the usual 4 feet 8½ inches gauge this load may generally be taken as equivalent to 1 ton for each foot in length of each line of way, or in engineering language, "one ton per foot run." Where a very heavy class of locomotives is in use 1¼ tons per foot run must be provided for. For small spans the distribution of the load as a locomotive passes is such that the above allowance is barely sufficient. For very small spans of 8 or 10 feet the maximum passing load is a little more than the weight on the driving axle of the locomotive, or say 14 tons.

§ 2. *Classification.*—Bridges are classed, according to the design of their superstructure, as *girders*, *arches*, and *suspension bridges*. A beam of wood crossing a stream, a brick arch, and a platform hung to a flexible wire rope are common examples of the three types. The essential distinction between the three types may be said to be, that in all forms of the suspension bridge the supporting structure (*i.e.*, the wire rope in the above example) is *extended* by the stress due to the load; in all forms of the arch the supporting structure (*i.e.*, the ring of bricks in the above example) is *compressed* by the stress due to the load; and in all forms of the beam or girder the material is partly extended and partly compressed by the flexure which it undergoes as it bends under the load,—thus when a beam of wood carrying a load bends, the upper side of the beam is thereby shortened and the fibres compressed, while the lower side of the beam is lengthened and the fibres extended.

Beams or *Girders* may be of various materials,—wrought iron, cast-iron, and wood being chiefly employed.

Arches may be of masonry, or they may be of wrought or cast iron or steel, in which case the compressed sector of a ring is usually a continuous and stiff structure.

Suspension bridges are made of wire ropes or of separate links of wrought iron or steel pinned together so as to form a chain. The metal beam, arch, or suspension bridge may be a continuous structure or an open frame; we shall also find that in some designs the several simple types are combined so as almost to defy classification.

Whatever design be adopted, the strength or efficiency to carry a given load depends on similar considerations. The designer selects that form of superstructure which the principles of statics show to be most desirable; he calculates the maximum stress which the load can produce on each part, and then so distributes his material that the maximum intensity of stress on every part shall be a definite fraction of the ultimate strength of the material. In metal structures, where the above principle can be very perfectly carried out, this fraction varies from one-sixth to one-third, according to the certainty with which the stresses and strength of the materials are known. In stone structures the engineer has greater difficulty in calculating the stresses on each part, and relies more on empirical rules based on long experience.

I. STRENGTH AND OTHER PROPERTIES OF MATERIALS EMPLOYED IN BRIDGES.

§ 3. *Classes of Stress.*—There are three kinds of stress, due to tension, compression, and shearing. Tension tends to cause failure by the extension or lengthening of the part strained; compression tends to cause failure by the crushing of the part strained; and shearing stress tends to cause failure by the sliding of one part of the piece across the other from which it is shorn off.

§ 4. *Tenacity, or Strength to resist Tension.*—When tension is applied to a rod or link of any material so as to be resisted equally by each element of any imaginary section in a plane normal to the direction of the pull, this section, which is called a *cross section*, is said to be subject to a stress of *uniform intensity*. This intensity p is equal to the quotient of the whole pull P divided by the area S of the cross section, or

$$1. \quad \ldots \ldots \quad p = \frac{P}{S}.$$

The ultimate strength of a rod subject to uniform stress is proportional to the section S, and the ultimate strength of the material is measured by the maximum intensity of stress which it can bear, or in other words, by the stress which the unit area of cross section can bear; for example, if the unit of force employed be the weight of a ton, and the unit of area the square inch, the strength of materials will be measured in tons per square inch, or by the number of tons which will just tear asunder a rod one inch square, great care being taken that the load is so hung on the rod as to bear equally on all parts of the cross section.

The following table gives in tons or lbs. per square inch the ultimate strength f_t of some of the materials used in bridges:—

TABLE I.—*Tensile Strength of Materials* $= f_t$.

Name of Material.	Tons per sq. inch.
Wrought Iron Plates	20 to 25
,, ,, Bars and Bolts	25 to 30
,, ,, Wire	30 to 45
Steel Plates	30 to 40
Steel Rivets	41 to 48
Steel Wire	50 to 100
Cast-iron	6 to 8
Red Pine	5·1 to 6·3
Larch	4 to 5·5
Oak	4·5 to 8·5
Teak	6 to 9
	Lbs. per sq. inch.
Brick (specimens of)	250 to 300
Basalt ,,	1000
Sandstone ,,	285
Common Mortar	10 to 50
Hydraulic Mortar	85 to 140
Roman Cement, 12 months old	46
Portland Cement, 7 days old	270
,, ,, 12 months old	350 to 470

The ultimate strength P_1 of a bar with the cross section S to resist a stress uniformly distributed over that section is given in terms of f_t by the expression—

$$2. \quad \ldots \ldots \quad P_1 = Sf_t.$$

Table I. gives some idea of the tensile strength of the materials, but for a full comprehension of the subject special treatises, or the article on the STRENGTH OF MATERIALS, must be consulted. No two specimens of any material ever give exactly similar results. The various brands of iron differ much; different specimens of steel differ still more; the strength is greater when the stress is applied along than when it is applied across the fibres. Iron or steel forged or drawn down to a small cross section is stronger than when in large masses; the material in small castings is stronger than in large castings run from the same batch of metal; the skin is stronger than the rest of a casting. The variations to be expected in timber are still greater than those in metals, and the values for stones or bricks must in each case be specially determined by experiment on the special kind to be used. These warnings are applicable to the subsequent tables, which give the strength of materials to resist other kinds of stress.

§ 5. *Strength to resist Crushing.*—The law given for tension applies to the compression of blocks, so that the strength f_c of a material to resist crushing may also be measured in tons per square inch. Thus the ultimate strength P_1 of a block of the cross section S, subject to a uniform stress, is given in terms of f_c the strength of the material per unit of section by the expression—

1. . . . . . . $P_1 = Sf_c$.

This equation is not applicable to blocks or struts of iron in which the ratio of the shortest side of the cross section to the length is less than about 1 to 5, nor to struts of timber in which this ratio is less than about 1 to 10. When this limit is passed the strut bends before failing (*vide* § 57), and whenever this occurs the essential condition that the stress shall be uniformly distributed no longer obtains. The flexure increases the stress on the inside of the curve assumed by the strut or pillar, and diminishes it on the outside; but the strut will fail as soon as *any part* of the cross section is subject to a crushing stress of greater intensity than f_c.

TABLE II.—*Ultimate Strength to resist Crushing* $= f_c$.

Name of Material.	Tons per sq. inch.
Wrought Iron	15 to 18
Cast-iron	37 to 65
Steel Plates	90
Red Pine	2·4 to 2·76
Larch	2·5
Oak	2·7 to 4·5
Teak	5·4
Bricks—	Lbs. per sq. inch.
Portland Cement, 3 months old	2460
,, ,, 9 ,,	3750
Gault Bricks	710 to 930
Best Whites	300
,, poor quality	60
Best Reds	490
Best Fire Bricks	930
Best Blue	1260
Stones (specimens)—	
Portland Stone (on bed)	2630
,, ,, (against bed)	2390
Bramley Fall (on bed)	5100
,, ,, (against bed)	2950
Yorkshire Landing (on bed)	5380
,, ,, (against bed)	5850
Granite	5500 to 11,000
Scotch Basalt	8300
Greenstone	17,200
Sandstones (ordinary)	3300 to 4000
Chalk	330 to 500
Béton	420 to 580

§ 6. *Strength to resist Shearing.*—Let a bar AD of any material be firmly supported on C, as shown in fig 1, and let a strong tool B, say of steel, descend upon it in the direction of the arrow, with force sufficient to sever the part D from the part A, so that the surface dividing the two parts is in the plane of the face of C. This tool is said to *shear* the bar, and the resistance which the bar opposes to this stress is called its strength to resist shearing. The tools practically used to shear are not quite square at the edge, and therefore *cut* slightly, but for true shearing the lower face ought to be square, and the tool should come down close to the support, so that the inner face of the tool slides on the outer face of the support.

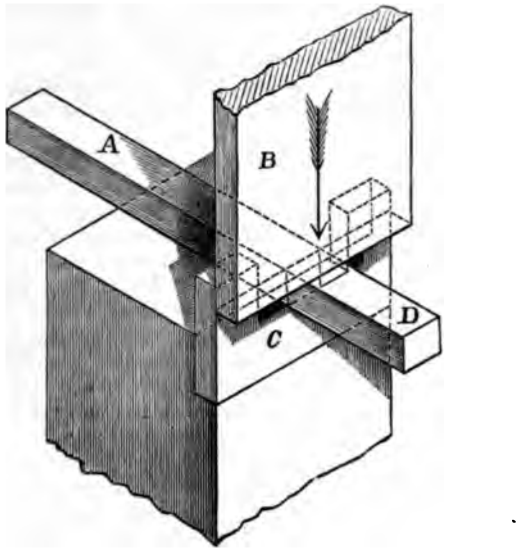

Fig. 1.

Fig 2. represents two iron links joined by an iron pin. If the links are pulled asunder the pin will be shorn at two places, A and B, and the whole section shorn will be twice the cross section of the pin.

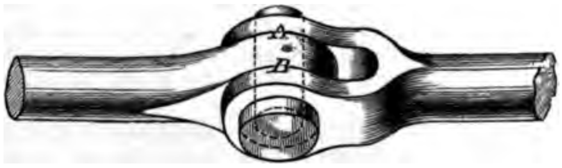

Fig. 2.

Fig 3 shows a joint where the pin would be shorn in four places—A, B, C, and D.

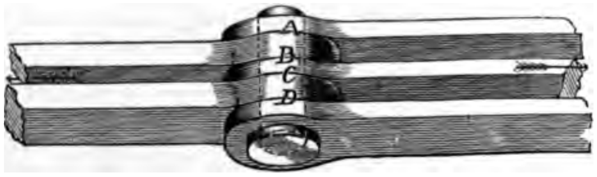

Fig. 3.

The strength of a piece of any material to resist shearing is usually assumed by engineers to be proportional to the cross section to be shorn through, and each material may consequently be said to have a certain shearing strength per square inch; in other words, the ultimate shearing strength of the material is the *intensity* of stress required to shear it asunder. If, therefore, P_1 be the ultimate strength of a bar of cross section S to resist shearing in *n* places, and if f_s be the ultimate strength of the material, we have the expression—

1. . . . . . . $P_1 = nSf_s$.

The assumption on which this equation is founded is not strictly correct; indeed, the actual shearing described does not correspond with any simple homogeneous stress, and the form of the cross section shorn through must exercise considerable influence on the strength of the piece to resist shearing. In a round pin the maximum intensity of shearing stress is $\frac{4}{3}$ of the mean intensity, and in a rectangular pin $\frac{4}{3}$ of the mean intensity. See STRENGTH OF MATERIALS.

The pins which join the links of suspension bridges, and the rivets which join the wrought iron plates of girder bridges, are subject to shearing stress, and the area to be shorn through must be made sufficient to bear the total shearing stress on that part of the structure. Wood is strong to resist tension, and would be much employed for ties but for the difficulty of taking hold of the ends of the tie in such a way that these ends shall not fail by being shorn.

Fig 4 shows the end of a balk of wood with a strap bolted to it. This strap would be torn off by the shearing of the wood along the dotted lines *ab* and *cd*, with a stress which would be much less than that required to overcome the tenacity of the wood; for if the dimensions of the balk of pine are 6 in. by 2 in. with an inch hole, its tensile strength will be 10 × 5 or 50 tons, while if the bolt be 1 inch diameter, and be placed 4 inches from one end, it could be torn out by shearing 16 inches of the wood; now each inch will only resist a shearing stress of say 600 lb, so that the bolt would be torn out with only 4·3 tons. Thus at least eleven such bolts, each 4 inches from its neighbour, and occupying 3 feet 8 inches in length of this balk, with an iron strap of corresponding length, would be required to render the full strength of the balk as a tie available. A similar difficulty is met with when timber is joined, as in fig. 5, where shearing would take place along the dotted lines AB or CD.

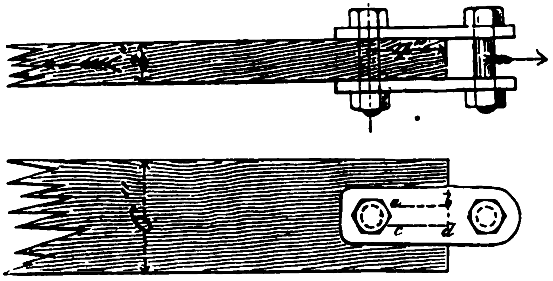

Fig. 4.

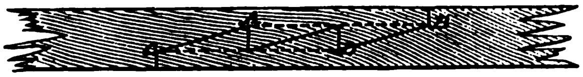

Fig. 5.

TABLE III.—*Ultimate Strength to resist Shearing* = *f*.

Name of Material.	Tons per sq. inch.
Cast-iron	14·5
Wrought Iron	22
Steel Rivets	30 to 36
Red Pine	·22 to ·35
Larch	·43 to ·75
Oak (British)	1

The strengths given here for wood are those obtained by testing the resistance to shearing along the grain.

The strength of wrought iron to resist shearing may be taken as practically equal to its strength to resist tension—an assumption which facilitates the design of rivetted and other joints.

The strength of steel to resist shearing is less than its strength to resist tearing in the ratio of three to four approximately. Rivets employed to joint steel plates require, therefore, to be larger or more numerous than those employed for iron plates of equal dimensions.

§ 7. *Elasticity.*—When a piece of any material is under tension or compression, it is lengthened or shortened by the stress, and the amount of extension or compression for the same length and stress varies with different materials. Approximately correct results are obtained by assuming that the extension or compression of a given piece of the material of uniform cross section, under a uniformly distributed stress constant throughout its length, is proportional to the length of the piece and to the intensity of the stress.

Let p be the intensity of the stress in tons per square inch, let l be the length of a specimen of a given material in any unit, and let e be the extension or compression observed in the same unit. Then the expression $\frac{pl}{e}$ is, on the above assumption, a constant quantity, and this ratio is experimentally found for many materials to be sensibly constant for stresses which do not approach the ultimate strength. This constant ratio E has received the name of modulus of elasticity, and is generally expressed in tons per square inch, this being the unit in which p is measured. Thomson and Tait (*Elements of Nat. Phil.*) call E the "measure of longitudinal rigidity," a better name than the modulus of elasticity. The actual extension or compression e of any piece of a structure is given by the expression—

$$1. \quad \ldots\ldots \quad e = \frac{pl}{E},$$

where p is in tons per square inch; e will be given in terms of the unit in which l is expressed.

TABLE IV.—*Modulus of Elasticity* = E.

Name of Material	Tons per sq. inch.
Wrought Iron	10,000 to 13,000
Iron Wire	11,500
Wire Ropes	6700
Steel Bars	13,000 to 19,000
Cast-iron	5250 to 10,000
Slate	5800 to 7100
Red Pine	650 to 850
Larch	400 to 600
Oak	535 to 780
Teak	1070

The modulus of elasticity is very generally assumed as equal for extension and compression, which is nearly true for wrought iron and steel under any stress to which these materials can be safely subjected. The principle of continuity shows that there can be no difference in the value of E for positive and negative stresses so long as these are small. The law according to which E varies as p changes is not accurately known for any material. When the stress is small the value of E appears to be more nearly constant in wrought iron than in cast-iron, but the change in the value of E corresponding to a change from a small stress to the ultimate stress is greater in wrought iron than in cast-iron. The values in the table cannot be depended upon to give very exact results, as the elasticity of different specimens of the same material varies considerably; the theory is wholly inapplicable when the breaking strain is approached; the engineer, however, seldom requires to calculate the extensions or compressions when the stress is even so great as one-third of the ultimate strength. For the same reason the engineer need seldom take account of the permanent set caused by stresses exceeding those within which the material may be considered perfectly elastic. B. D. Stoney, in his work on the theory of strains on girders, gives an excellent summary of what is known experimentally concerning *set* and the modulus of elasticity.

§ 8. *Strength to resist Tension or Crushing when the Stress is not Axial.*—When the resultant of a stress passes through the centre of surface of the cross section of the piece of a structure, and is normal to the cross section, the stress is called *axial*, and it is usually assumed that this stress will be borne uniformly by all the elements into which the surface of the cross section may be conceived as divided. This is not necessarily true, but it is approximately true for the forms usually employed by engineers. When the stress is not axial it cannot be considered, even approximately, as uniformly distributed; the greatest intensity of stress will occur towards that edge of the cross section which is nearest to the point where the resultant meets the plane of the cross section. The following considerations allow the maximum intensity of stress to be approximately calculated in most of the cases which are practically met with. 1. Let the cross section (fig. 6) have an axis of symmetry XX_1, and let YY_1 be an axis passing through the centre of gravity of the surface at right angles to XX_1; the axis YY_1 will be called the *neutral axis*. 2. Let the resultant stress pass through some point A in the axis XX_1, at a distance x_0 from the axis YY_1. 3. Let the material be such that its modulus of elasticity is constant under all the intensities of stress which result from the given total stress; then calling the whole force P, the area of the cross section S, the mean intensity of stress p_0, the maximum intensity of stress p_1, the maximum distance of any point of the cross section from the axis x_1, and representing by I the moment of inertia (*vide* § 9) of the surface of the cross section round the axis YY_1, we observe—first, that

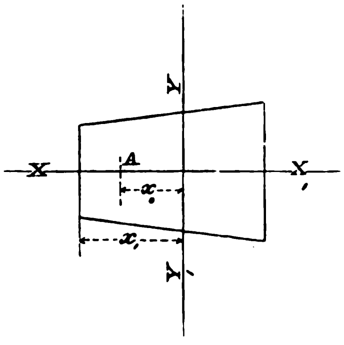

Fig. 6.

$$p_0 = \frac{P}{S};$$

and secondly, that the non-axial force may be assumed to produce a uniformly varying stress, the maximum intensity of which will occur at the distance x_1 from the neutral axis on the same side as x_0 is taken. This uniformly varying stress is equivalent to a uniformly distributed stress of the intensity p_0 and a couple of the moment aI, where a is the constant rate of increase of the stress. We also know by the principles of mechanics that the force P applied at a distance x_0 from the centre might be replaced by an equal force P at the centre and a couple of the moment Px_0. Hence we have—

$$1. \quad . \quad . \quad . \quad . \quad Px_0 = aI, \text{ or } a = \frac{Px_0}{I}.$$

But the maximum intensity of stress due to the couple will be $x_1 a$, *i.e.*, $\frac{Px_0x_1}{I}$, and the maximum stress p_1 may be considered as consisting of two parts,—first, the mean intensity of stress p_0, and secondly, the maximum stress due to the couple. Hence we have—

$$2. \quad . \quad . \quad . \quad p_1 = p_0 + \frac{Px_0x_1}{I} = p_0\left(1 + \frac{x_0x_1S}{I}\right).$$

This equation shows that the maximum intensity of stress increases very rapidly as x_0 increases, and it must be borne in mind that the ultimate strength of any member of a structure is determined, not by the mean stress, but by the maximum stress on any part, for when the part most strained yields the structure is weakened thereby, and this failure must continue to extend until the whole yields.

§ 9. *Note on the Value of* I, *the Moment of Inertia of the Cross Section.*—Suppose the cross section divided into a very large number of rectangles, so small that the distance x of the centre of each from the neutral axis may be taken as sensibly expressing the distance of every part of the rectangle; then I is the sum of the products of the areas of all the elementary rectangles each multiplied into the square of its distance from the neutral axis; or calling the area of the elementary rectangle $\Delta x \Delta y$,—

$$1. \quad . \quad . \quad . \quad . \quad . \quad I = \Sigma x^2 \Delta x \Delta y.$$

The subjoined table gives the values of I for some simple geometrical forms; the axis in all cases passes through the centre of gravity of the surface.

TABLE V.—*Moment of Inertia* = I.

Surface.	Neutral Axis.	I.
Circle, radius r	Diameter	$\frac{1}{4}\pi r^4$
Square, side d	Parallel to one side	$\frac{1}{12}d^4$
Square, side d	Diagonal	$\frac{1}{12}d^4$
Rectangle with sides d and b	Parallel to b	$\frac{1}{12}bd^3$
Triangle, base b, height d	Parallel to b	$\frac{1}{36}bd^3$

Whenever a cross section can be conceived as obtained by the addition or subtraction of one surface to or from another, both surfaces having a common neutral axis, the value of I for that cross section is got by adding or subtracting the value of I for one surface to or from that for the other; thus for a ring surface, with external and internal radii r and r_1, the value of I is—

$$\tfrac{1}{4}\pi(r^4 - r_1^4).$$

The value I_1 of the moment of inertia of any plane surface S round an axis in its own plane parallel to the neutral axis, and at a distance x_0 from that axis, is given by the equation—

$$2. \quad . \quad . \quad . \quad . \quad . \quad I_1 = I + Sx_0^2,$$

where I is the moment of inertia of the surface round a neutral axis, *i.e.*, round a parallel axis passing through the centre of gravity of the section. We can thus obtain the value of the moment of inertia I of complicated cross sections whenever we can divide these into rectangles, circles, or triangles; for then, calling s_1, s_2, s_3, &c., the surfaces of each elementary part, x_1, x_2, x_3 the distances of the centre of each part from the neutral axis of the whole cross section, and $I_{\prime}$, $I_{\prime\prime}$, $I_{\prime\prime\prime}$ the moments of inertia of each element calculated round its own neutral axis, we have the moment of inertia of the whole round its neutral axis—

$$3. \quad . \quad . \quad . \quad . \quad . \quad I_0 = \Sigma I^2 + \Sigma s x^2.$$

§ 10. *Specific Gravity of Materials.*—In order to calculate the load due to the superstructure of a bridge, and the stability due to the weight of the abutments and anchorages of arches and suspension bridges, it is necessary to know the specific gravity of the materials employed—specific gravity being for the purposes of the present article defined as the weight of the material in lbs. per cubic foot. The following are the most useful numbers:—

TABLE VI.—*Weight per Cubic Foot of Different Materials.*

Name of Material.	Weight of cubic foot in lbs.
Water (pure) at 39°·4. Fahr.	62·425
Basalt	187·3
Brick	100 to 135
Brickwork (ordinary)	112
Chalk	117 to 174
Clay	120
Granite	164 to 172
Limestone and Marble	169 to 175
Masonry	116 to 144
Mortar	86 to 119
Mud	102
Sandstone	130 to 157
Sand (damp)	118
Sand (dry)	88·6
Asphalt	156
Concrete (ordinary)	119
Concrete in cement	137
Earth	77 to 125
Slate	175 to 181
Trap	170
Cast-iron (average)	444
Wrought Iron (average)	480
Red Pine	30 to 44
Larch	31 to 35
Oak (European)	43 to 62

§ 11. *Change of Length due to Change of Temperature.*—The change in the dimensions of structures due to a change of temperature exercises a material influence on the durability and strength of structures, and must not be lost sight of in the design of any bridge of more than common dimensions. The following short table gives the coefficient by which the length of a bar of each material measured in any unit must be multiplied to obtain the increase in length (in the same unit) resulting from a rise of temperature of 1° centigrade.

TABLE VII.—*Coefficient of Linear Expansion per degree cent.*

Cast-iron.	Wrought Iron.	Stone (paving or granite.)	Sandstone (Craigleith).
0·000011	·000012	·000008 to ·000009	·000012
Slate (Penryn).	**Brick (best Stock).**	**Brick (Fire).**	**Dry Deal, in direction of grain.**
·00001	·0000055	·000005	·0000043

II. BEAMS OR GIRDERS.

§ 12. *External Forces.*—The beams or girders of bridges are subject to vertical loads, and they are supported by vertical reactions at piers or abutments. The sum of the loads is, therefore, necessarily equal to the sum of the reactions at the points of support; or calling P and P_1 the weights borne by two abutments (when the girder has no other support), and w_1, w_2, w_3, &c., the loads on various parts of the girder, we have—

1. . . . . . $P + P_1 = \Sigma w$.

Let L be the distance between the points of support, fig. 7,

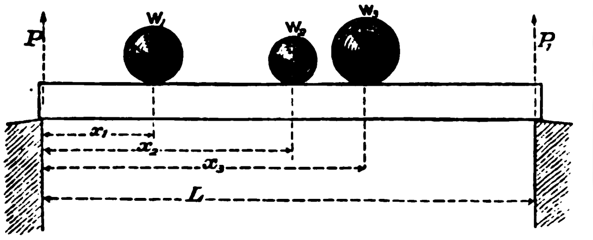

Fig. 7.

and let x_1, x_2, x_3 be the distances of the several loads w_1, w_2, w_3 from the abutment bearing the weight P; then, taking moments round the point of support at the above abutment, we have the upward reaction at the pier × span = the sum of the products of each weight into its distance from the point of support, or

2. . $P_1 L = \Sigma wx$, and similarly $PL = \Sigma w\,(L - x)$.

When the distribution of the loads is known, equation 2 gives the weight borne by each abutment. Applied to the case of a single load W rolling from end to end of a beam, calling x the distance of the load from the abutment supporting the weight P, equation 2 gives

3. . . . $P_1 = \frac{Wx}{L}$ and $P = \frac{W(L-x)}{L}$.

Applied to the case of a uniform advancing load, such as a railway train gradually covering the whole beam (fig. 8),

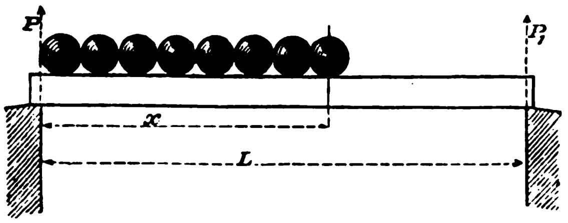

Fig. 8.

calling x the distance covered by the train measured from the pier bearing load P, and w the weight of the load per unit of length, equation 2 gives

4. . . . $P_1 = \frac{wx^2}{2L}$ and $P = wx\,\frac{2L-x}{2L}$.

These equations express the fact that the beam, as used in a bridge, is as a whole in equilibrium under a system of parallel vertical forces which may be called the *external forces*, and which are all determinate so soon as the distribution of load and the span are given.

§ 13. *Internal Forces.*—The external forces call into play certain internal forces. A beam of given design will be properly *proportioned* if each part has just those dimensions which are suitable to bear the maximum *internal* stress which any distribution of load can bring to bear upon it; and the beam will be properly *designed* if its form is such as to enable it to bear the load with the least possible quantity of material. A complete analysis of the internal forces in a loaded beam would in any case be exceedingly difficult and with most designs impossible, but it is found by experiment that a beam will bear a given load if we provide strength enough to resist the *horizontal* components of those internal forces which tend to extend or compress the beam in the direction of its length, and if we also provide against the *vertical* components which tend to shear the beam across in planes perpendicular to the longitudinal axis of the beam. The nature of the horizontal and vertical forces due to the elastic reaction of the material will be understood by reference to figs. 9, 10, 11.

Let a model be made of four stiff light frames of wood A, A_1, A_2, A_3, each say 18 inches deep, 18 inches long, and 6 inches wide (fig. 9). Let these be connected with one another by small cylinders of india-rubber, $a\ b\ c\ d$, $a_1\ b_1\ c_1\ d_1$, and $a_2\ b_2\ c_2\ d_2$. These cylinders must be so attached to the frames as to be capable of resisting both extension and compression. The whole structure will now have somewhat the appearance of a beam, but if it is placed between two supports Q, N, it will be found unsuited to carry even its own weight, because the middle frames will tend to slip down between the two others.

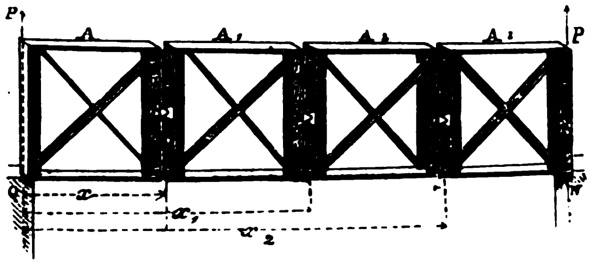

Fig. 9.

This tendency will be still better seen when a load is placed on the imperfect beam. Fig. 10 shows the tendency to shear off the loaded from the unloaded part of the beam. The frame A_2 is forced down below the frame A_3 by a shearing stress resisted by the india-rubber in a very

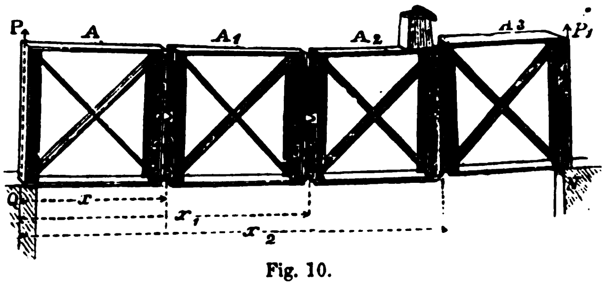

Fig. 10.

imperfect manner. To make the frame into a true beam, this tendency of the loaded frame to slip through between the others must be counteracted by tongues, T, T_1, T_2, projecting from one frame and working in a groove in the neighbouring frame (*vide* § 19). Each tongue should be made so that it does not abut against the bottom of the groove, and is thus incapable of resisting any horizontal force—it must neither prevent the whole beam nor any part of it from being extended or compressed longitudinally.

The structure will now be found capable of carrying weights as a beam. It deflects or bends as in fig. 11 under the action of a load on A_1; all the pieces of india-rubber above the centre of the beam are then compressed; all those below the centre are then extended; and at the sections between the frames the horizontal internal forces are wholly met by the elastic reaction due to these horizontal extensions and compressions.

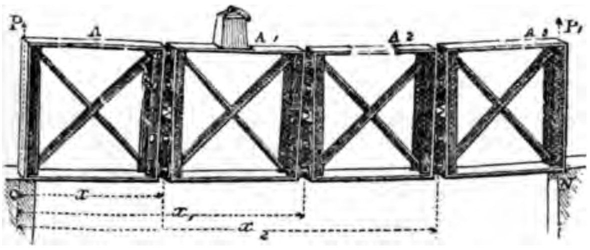

Fig. 11.

At any one section, say at a distance x_1 from the support Q, the pieces c_1 and d_1 are just as much compressed as the pieces $b_1\,a_1$ are extended; the equal and opposite parallel resultants of these forces consequently constitute a couple, and the moment of this couple must be equal to the moment of the couple tending to bend the beam at this section, or to what is called the *bending moment*; now the *bending moment* in this simple case is due to the upward vertical reaction P_1 at N and the equal downward force with which the frame A_1 bears on the tongue T_1; for it is clear that, neglecting the weight of the frame, if a weight W on frame A_1 is borne by two forces P and P_1 at the two piers, the tongue next N must also bear a vertical force P_1 and the tongue next Q a vertical force P. The stresses borne by the india-rubber pieces are exactly the same as if the frame A_1 were firmly held, and a vertical force P_1 applied to pull up the right hand part of the beam, while the tongue T_1 acted as a hinge; the moment tending to turn the right hand part round in a left-handed direction would be $P_1\,(L-x_1)$. This moment is resisted by the elasticity of the india-rubber, which must exert an equal and opposite moment round the same point. Calling s_1, s_2, s_3 the sectional areas of the pieces of rubber, and y_1, y_2, y_3 their distances from the axis where the section is neither extended nor compressed, and p_1, p_2, p_3 the intensity with which each piece is strained, the moment due to the elasticity of the pieces of rubber tending to turn back the left hand part of the beam in a right-handed direction will be Σpsy. Now, if the modulus of elasticity of the rubber is constant, the stresses p_1, p_2, p_3 will be proportional to their distance from the unstrained axis; thus if b_1 is 18 inches from the axis and c_1 only 9 inches, b_1 being shortened twice as much as c_1, the stress on b_1 will be twice that on c_1, and calling a the stress at unit distance from the axis we have $p_1 = ay_1$ and $p_2 = ay_2$, $p_n = ay_n$, so that we may write $\Sigma psy = a\Sigma sy^2$ as the expression for the moment of the elastic forces. Hence equating the bending moment and the moment of the elastic forces, we have—

$$1. \quad . \quad . \quad . \quad . \quad P_1(L-x) = a\Sigma sy^2 .$$

From this equation when the load is given we may determine a, and hence the intensity of the stress $p = ay$ at any distance y from the axis. If this intensity is less than the safe stress for the material, the beam is, at the section considered, strong enough to bear the load so far as the horizontal extending and compressing forces are concerned.

Thus if the dimensions of the beam be those given above and it be supported so that L may be 6 feet and the distance x_1 2 feet 3 inches, the distance y of the outermost piece of rubber from the unstrained axis 8 inches, the weight 50 lb, and the section of the rubber in each row 2 inches (two cylinders side by side, each with a section of 1 inch) we shall have as the numerical values in the above equation, neglecting the weight of the frame itself (P_1 being nearly = 18·7), $18{\cdot}7 \times 45 \text{ in.} = a\,(4 \times 8^2 + 4 \times 4^2)$ from which $a = 2{\cdot}64$; then the force supported by each inch of either of the rows of rubber a_1 or d_1 will be $p = 8 \times 2{\cdot}64 = 21{\cdot}12$ lb; the stress on each of the inner rows will be half this amount; the same equations give the load which (so far as that particular section is concerned) can safely be placed on the frame A_1 consistently with a given stress per square inch on the rubber. The strength required in the tongue T_1 is still more easily found, the stress tending to shear it off is P_1, and it must have a sufficient cross section to bear that shearing stress. Similar reasoning would allow us to calculate the strength of our beam at either of the two other sections where the india-rubber pieces and tongues are placed. The general relation between the external and internal forces in any beam is similar to that illustrated by the model; at any section the moment due to the elastic forces must balance the moment due to the external forces tending to bend the beam at that section. The problem, therefore, of determining the strength of a beam at any section resolves itself, so far as the horizontal forces are concerned, into finding expressions for these two moments and equating them. The equation thus stated will give the maximum horizontal stress thrown on the material of a given girder by a given load, or it will give the maximum load on a given girder consistent with a safe stress on the material, or if, as is generally the case in bridges, the load and maximum safe stress on the material are given, the equation will allow us to fix the dimensions required for the cross section of the beam so far as the horizontal forces are concerned. The provision to be made for resisting the shearing stress for which the tongues are required in the model will be explained in § 19.

In any solid beam the stresses do not divide themselves into horizontal and vertical components. This division is made by the engineer to simplify his calculations. In the beam the actual stress at any point will be the resultant of the horizontal stress (borne by the india-rubber in the model) and the vertical stress (borne by the tongue in the model). The diagram, fig. 12, shows the direction of the resultant stress at each point of a beam of rectangular cross section. The curves are called the lines of principal stress.

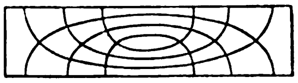

Fig. 12.

§ 14. *Bending Moment and Moment of Elastic Forces or Moment of Stress.*—The bending moment for a given section of a given beam under a given distribution and magnitude of load is the sum of the moments, taken relatively to the section, of all the external forces acting on one of the two segments into which the section divides the beam. It is a matter of indifference which segment is considered, but the moment on one segment will be positive and that on the other negative. Let x be the distance of the section from the left hand abutment Q of any beam of span L (fig. 13). Let P be the load borne by abutment Q, and P_1 the load borne by abutment N. Let the load on the beam, including the weight of the beam, be divided into any number of equal or unequal parts. Let those loads which lie to the left of the section be called w_1, w_2, w_3, &c., and let the distances of their centres of gravity from the section be called l_1, l_2, l_3. Let those loads which are to the right of the section be called u_1 u_2, u_3, and let their distances from the section be called r_1, r_2, r_3 &c. Let the bending moment at the section be called M, then by the above definition

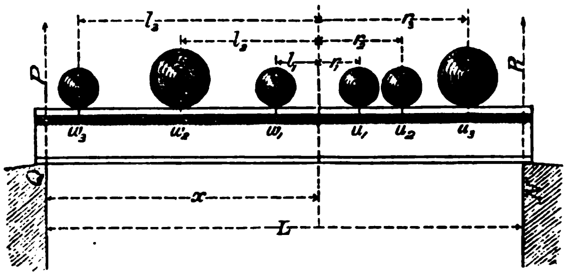

Fig. 13.

1 . $M = Px - \Sigma wl = \Sigma ur - P_1(L - x)$.

This bending moment is resisted at the section by the elastic forces in the beam called into play by compression at the top and extension at the bottom, or, in other words, called into action by the stresses on the material. Limiting our consideration to those cross sections which have a vertical axis of symmetry, and to those materials which have a constant modulus of elasticity for tension and compression, it is easy to find a general expression for the moment of the elastic forces, which will hereafter be designated by the letter μ. Let the horizontal axis ZZ_1 (fig. 14) be a line along which the material is unstrained,—all the portions above this line being compressed, and all the portions below this line being extended. Conceive the section divided into little elementary surfaces, the area of each being equal to the product of the little elementary length Δy into the little elementary length Δz, the force exerted by the elasticity of this element will be proportional to its area $\Delta y \Delta z$ and to its distance y from the axis ZZ_1, for the longitudinal extension or compression is directly proportional to this distance; then calling a, as before, the rate at which the stress increases with the increase of y, or in other words the intensity of the stress at the unit distance from ZZ_1, we shall have for the force exerted by each element the expression $ay\Delta y\Delta z$. Now, as the section is not moved as a whole along the beam in either direction, we must have $\Sigma ay\Delta y\Delta x = 0$, that is to say, the sum of the positive must be equal to the sum of the negative forces. Let the constant quantity $a\Delta y\Delta x$ be conceived as the weight w of a thin plate of the area $\Delta y\Delta x$, the moment of this weight relatively to the axis will be wy, but as we have the sum of the moments $\Sigma wy = \Sigma ay\Delta y\Delta x = 0$, the axis ZZ_1 round which these moments are taken must pass through the centre of gravity of the section. This axis is called the *neutral axis* of the beam (*vide* § 18). The expression for the force exerted by each element being $ay\Delta y\Delta x$, the moment of this force, or, in other words, for the moment due to the stress, is given by the expression $ay^2\Delta y\Delta z$; then for the moment μ of all the forces round ZZ_1 we have—

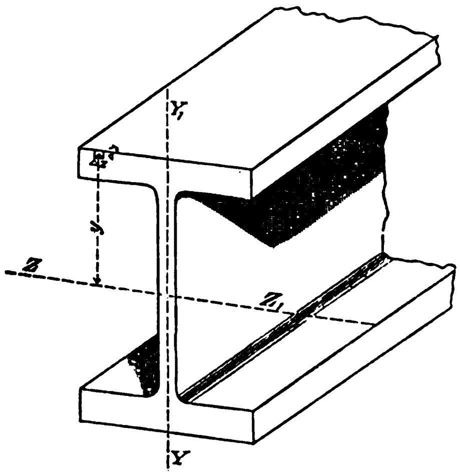

Fig. 14.

2. . . . . $\mu = a\Sigma y^2\Delta y\Delta z = aI$,

where I is the moment of inertia of the surface of the cross section. This expression μ is Rankine's moment of resistance to flexure; it may also be called the moment of the elastic forces, or, as suggested by Professor Kennedy, the moment of stress.

The greatest intensity of stress p_1 occurs at the greatest distance from ZZ_1, or at the distance $\frac{1}{2}d$ if the depth of the beam be called d, and the axis ZZ_1 be equidistant from top and bottom. In this case we have—

$$p_1 = \frac{ad}{2}, \text{ or } a = \frac{2p_1}{d};$$

hence we may write for sections with two axes of symmetry, and for materials having equal strengths and constant moduli of elasticity under tension and compression—

3. . . . . $\mu = 2\,\frac{p_1 I}{d}$.

Now, the general condition of equilibrium between the external forces and the elastic forces at the section is simply $M = \mu$, hence when the beam does not break we have also—

4. . . . . $M = \frac{2p_1 I}{d}$.

Whenever the amount and distribution of load on a given girder is known the bending moment M is to be calculated from equation 1, and then equation 4 allows us to calculate p_1; or if we know the value of f, the ultimate strength of the material to resist tension and compression, this equation enables us to find what moment M_1 is required to break the beam at that section, and therefore what load distributed in a given way the beam can bear; thus we have—

5. . . . . $M_1 = \frac{2fI}{d}$.

In a beam of uniform strength the value of $\frac{2fI}{d}$ will at every section be equal to the value of M_1, the moment due to external forces at that section.

When the material is not equally strong to resist tension and compression, let f_t and f_c be the two strengths (per unit of cross section), but let the modulus of elasticity be assumed constant as before. Then, as above, the unstrained axis will be the axis passing through the centre of gravity of the section, and the intensity of stress at any distance y above or below that axis will be ay; let y_c be the distance of the uppermost element of the cross section from the horizontal axis, and let y_t be the distance of the lowermost element. Then the greatest stress will occur at the top if y_c be greater than y_t, but at the bottom if y_t be greater than y_c; since, however, the material is not equally strong to resist tension and compression, it does not follow that it will give way where the stress is greatest, and the beams will yield first at that edge where the ratio $\frac{y}{f}$ is greatest.

We must, therefore, on a beam of this kind ascertain whether $\frac{y_c}{f_c}$ or $\frac{y_t}{f_t}$ is the greater, and replace $\frac{d}{2}$ in equations 3 and 4 by the value of y in the larger of the two expressions. Having selected y and f in this manner, we have—

$$6. \quad . \quad . \quad . \quad . \quad . \quad . \quad M_1 = \frac{fI}{y}.$$

§ 15. *Modulus of Rupture.*—If the above hypothesis of a constant modulus of elasticity in a given material under both descriptions of stress, and under stresses of all magnitudes, were accurate, we should require no fresh experiments to determine the values of f_c and f_t; these would be already known from direct experiments on tension or compression. For both wrought and cast iron beams of the maximum strength, such as will be described hereafter, experiment gives results closely in accordance with strengths calculated by equation 6; and for wrought iron beams the hypothesis appears to be approximately true when tested by experiment with any form of beam. But it is usual, in calculating the strengths of beams or bars of simple rectangular or circular cross section, to assume that y is equal to $\frac{1}{2}d$ and to employ equation 5 instead of equation 6. The imperfection of the theory is then to some extent corrected by determining f from direct experiments on solid rectangular bars. The value of M_1, I, and d being known, f is calculated from equation 5, and the number thus determined has received the name of modulus of rupture.

TABLE VIII.—*Modulus of Rupture* = f (*Rankine*).

Name of Material.	Lbs. per sq. inch.
Wrought Iron Plate Beams	42,000
Cast-iron Solid Bars	33,000 to 43,000
Fagersta Steel (Kirkaldy)	110,000 to 191,000
Red Pine	7100 to 9540
Larch	5000 to 10,000
Oak (British and Russian)	10,000 to 13,600
Indian Teak	14,770
Sandstone	1100 to 2360

Experiments on the modulus of rupture have generally been made by hanging weights at the centre of a rectangular bar, supported at both ends, and increasing the weights till the bar breaks. Then let b and d and l be the breadth, depth, and length of the bar in inches, and W the breaking weight. M is a maximum at the centre, and, neglecting the weight of the bar, is equal to $\frac{1}{4}Wl$; substituting the value of I for a rectangle in equation 3, we get

$$\mu = \tfrac{1}{6}fbd^2,$$

and equating M and μ we have—

$$1. \quad . \quad . \quad . \quad . \quad . \quad . \quad f = \frac{3Wl}{2bd^2},$$

or, if the span is measured in feet and called L_1, while b and d are measured in inches, we have—

$$2. \quad . \quad . \quad . \quad . \quad . \quad . \quad f = 18\,\frac{WL_1}{bd^2}.$$

Hence the modulus of rupture is stated by Rankine to be "eighteen times the load required to break a bar of 1 inch square, supported at two points 1 foot apart, and loaded in the middle between the points of support."

The use of a modulus of rupture determined by experiment on a special form of beam is not based on any satisfactory principle. The employment of this modulus is an imperfect means of correcting a defective theory. A different value is found for f when bars of different sizes or cross sections are tested. Even the same bar broken in different ways will give a sensibly different value for f. The use of this modulus is, however, convenient when great accuracy is not required.

§ 16. *Expressions for the Bending Moment caused by Special Distributions of Load.*—From equation 1, § 14, it is easy to deduce the following values for the bending moments in beams subject to various simple distributions of load:—

Case 1. Let a single load W be placed at the centre of a beam; let M_x be the moment at any section taken at the distance x from the nearest pier; let M_c be the moment at the centre of the span, we have—

$$1. \quad . \quad . \quad . \quad . \quad . \quad M_x = \tfrac{1}{2}Wx,$$

$$2. \quad . \quad . \quad . \quad . \quad . \quad M_c = \tfrac{1}{4}Wl.$$

M_c is the maximum moment at any section of the beam.

Case 2. Let a single load W be placed at the distance x_1 from the nearest pier; let M_{x_1} be the moment at the section where the load is applied—

$$3. \quad . \quad . \quad . \quad . \quad . \quad M_{x_1} = W\frac{lx_1 - x_1^2}{l}.$$

M_{x_1} is the greatest moment which the load at that place produces at any part of the beam, and increases as the load rolls from the end to the centre in proportion to the product $(l - x_1)\,x_1$ of the parts into which the load divides the beam.

Case 3. Let the load be uniformly distributed along the beam so that each unit of length bears an equal load $w = \frac{W}{l}$; when the unit of length adopted is the foot, the quantity w is called the *load per foot run.* Let M_c and M_x be as before the moments at the centre of span, and at any distance x from the nearest pier, we have—

$$4. \quad . \quad . \quad . \quad . \quad . \quad M_c = \tfrac{1}{8}wl^2,$$

$$5. \quad . \quad . \quad . \quad . \quad . \quad M_x = \tfrac{1}{2}w\,x(l - x).$$

When a uniform passing load, such as is approximately represented by a train of locomotives, of length at least equal to the span of the bridge, comes on to a bridge at one end and passes over to the other, gradually covering the whole span, the bending moment reaches a maximum for all sections when the bridge is wholly covered. The bending moment at any section for a combination of loads is the sum of the moments at that section due to each load taken separately. When many different distributions of load have to be provided for engineers are in the habit of representing the bending moment for each load by a line, the ordinates of which are the bending moments at the sections, and the abscissæ the distances of the several sections from one point of support. The lines having been drawn for the several loads, it is easy by superposition to find the bending moment due to the combination, and thus to pick out for each section of the girder the maximum bending moment which any combination gives. Figs. 15, 15*a*, 15*b*, and 15*c* show diagrams giving the curve of bending moments for some simple distributions.

Fig. 15 shows the line of bending moments when a single load of 10 tons hangs at the centre of a span of 50 feet. The vertical ordinates measured on the vertical scale give the bending moments at each section. Fig. 15*a* shows the curve of bending moments for a uniformly distributed load

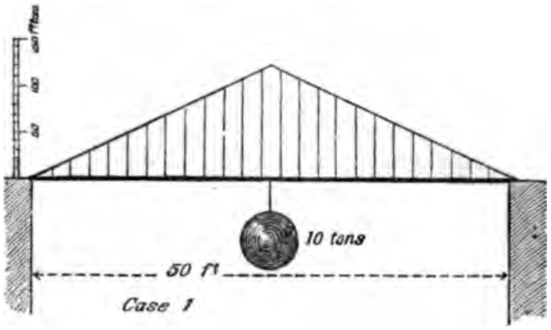

Fig. 15.

of 1 ton per foot run on a span of 50 feet. The upper curve in fig. 15*b* shows the curve of bending moments when the loads of case 1 and 2 both occur at once. This curve is obtained by adding the ordinates in case 1 to those in case 2. Fig. 15*c* shows the four separate lines of bending moment for four separate weights, and the broken upper line is the line of bending moments for the case when the four loads all rest at once on the beam; it is got by simply adding at each point of the span the four ordinates due to the four loads considered separately. The curve in fig. 15*a* and the curve ABCD in fig. 15*c* are parabolas.

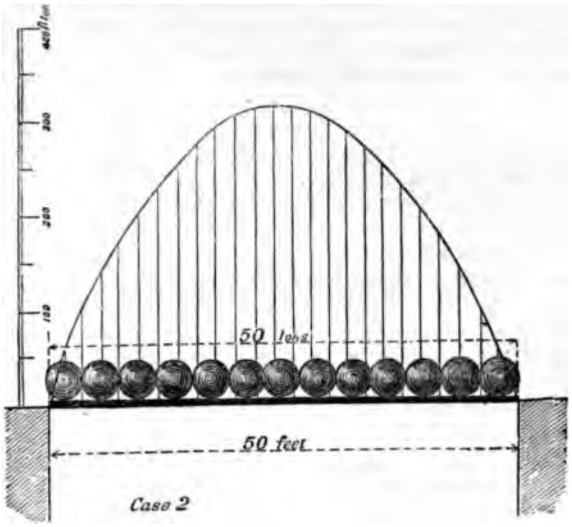

Fig. 15*a*.

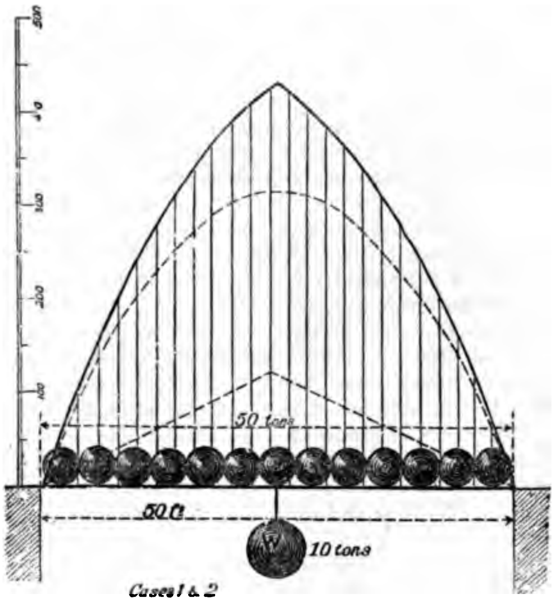

Fig. 15*b*.

The bending moment at any section is reckoned as *positive* when the external forces on the beam tend to turn the right hand side of the beam in a left-handed direction (or in the direction opposite to that followed by the hands of a watch), in other words, when the bending moment tends to bend the beam downwards between the supports. We shall hereafter see that in beams with more than two supports the bending moment is at places *negative*, tending to bend the beam up; the curve of bending moments is drawn below the datum line where the moments are negative. Fig. 25*a* gives an example of a curve of bending moments of this class.

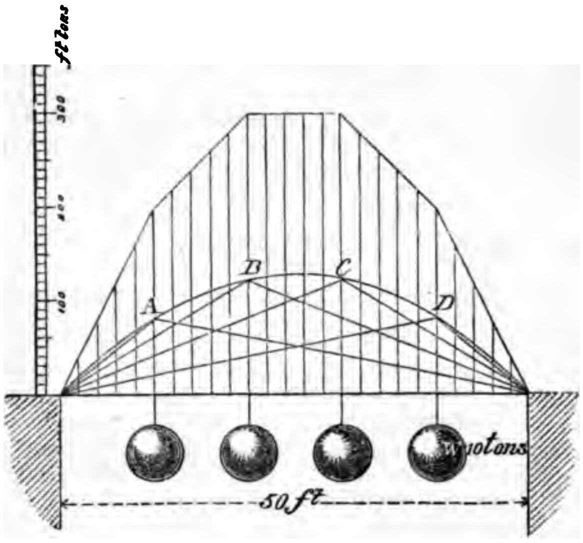

Fig. 15*c*.

§ 17. *Moment of Elastic Forces.*—On the hypothesis above stated, viz., that for any one material the modulus of elasticity may be taken as constant for all stresses, and assuming that our investigation is to be confined to those cases in which the cross section of the beam has a vertical axis of symmetry, and in which the centre of gravity lies at a point equidistant from the top and bottom of the beam, the general equation—

$$\mu = a\mathrm{I} = 2\,\frac{p_1\mathrm{I}}{d},$$

(3, § 14), allows simple expressions of the value of μ to be obtained for all practical cases by substituting for the ratio $\frac{2\mathrm{I}}{d}$ its value in terms of the dimensions of the cross section. Thus, for a rectangle of the depth d and breadth b,

$$\frac{2\mathrm{I}}{d} = 2\,\frac{bd^3}{12d} = \frac{bd^2}{6} = \frac{\mathrm{S}d}{6},$$

where S is the area of the cross section. The following are the values for the commonest forms of cross sections.

TABLE IX.—*Values of* $\frac{2\mathrm{I}}{d}$ *for various Cross Sections.*

Form of Cross Section.	$\frac{2\mathrm{I}}{d}$	
	In Terms of Linear Dimensions of the Cross Section.	In Terms of the Area of the Cross Section (S) and the Depth.
Square, side b	$\frac{b^3}{6}$	$\frac{\mathrm{S}b}{6}$
Rectangle, breadth b depth d	$\frac{bd^2}{6}$	$\frac{\mathrm{S}d}{6}$
Circle, radius r	$\frac{\pi r^3}{4}$	$\frac{\mathrm{S}d}{8}$
Hollow circle, external radius r, internal r_1	$\frac{\pi(r^4 - r_1^4)}{4r}$	$\frac{\mathrm{S}d}{4}$ when r_1 differs little from r.
Hollow rectangle, internal depth d_1, breadth b_1	$\frac{bd^3 - b_1d_1^3}{6d}$	$\frac{\mathrm{S}d}{2}$ when moment of web is neglected, and S is taken as area of cross section of flanges only, and d_1 differs little from d.

Substituting the modulus of rupture f (Table VIII.), for p_1 in equation 3, § 14, we can from the above values of $\frac{2\mathrm{I}}{d}$ calculate the ultimate strength of any given cross section to resist any given bending moment. Thus, if we wish to know what breadth we must give a bar of wrought iron 3 inches deep, supported at points 3 feet apart, so that it may break with 2000 ℔ at the centre, we find the maximum bending moment from equation 2, § 16; and we find the value of μ from equation 3, § 14, by the help of table IX. Then equating M and μ, we have—

$$1. \quad . \quad . \quad . \quad . \quad . \quad \tfrac{1}{4}\mathrm{WL} = \tfrac{1}{6}f\,bd^2\,,$$

from which $b = \frac{6\mathrm{WL}}{4d^2f} = \frac{6 \times 2000 \times 36}{4 \times 9 \times 42000} = {\cdot}286\,.$

Since the strength of the beam is directly proportional to the values of $\frac{2\mathrm{I}}{d}$, table IX. shows us how to dispose of the material in the cross section so as to obtain the maximum strength. The expression for a rectangle shows that the strength is proportional to the breadth of a beam, but to the square of its depth; and therefore, as appears in the second column, for the same quantity of material the strength of a rectangular beam increases simply as the depth, so that a deep narrow beam is preferable to a square or to a broad and flat one. The circular cross section is weak compared even with the square (the ratio for the same quantity of material is ·846 to 1). The hollow tube is stronger than the thin rectangular plate of the same depth and cross section, but clearly the material is much best applied when wholly used in the form of two thin flat plates, separated from one another by a web of the maximum depth d which can be practically allowed. Thus the hollow rectangle is the form preferred for large girders, the material intended to resist the bending moment being placed in the top and bottom members of the girder, and kept apart by vertical webs, which add somewhat to the moment μ, but which are chiefly employed to resist shearing stress, as will be presently shown.

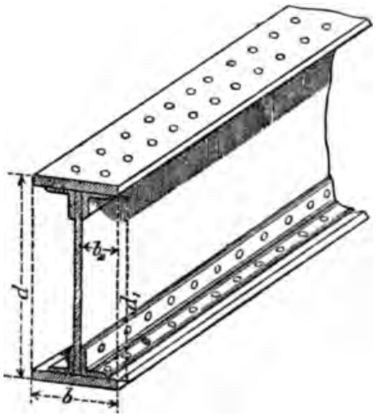

Fig. 16.

The I section, fig. 16, is that employed for small girders; the moment of its elastic forces is exactly the same as that of a hollow rectangle, having the same values for b, d, and d_1, and having a value for b_1 equal to $2b_2$ in fig. 16 It is usual to neglect the strength to resist bending moment given by the vertical web, and to design the girder so that the top and bottom plates are alone sufficient to meet this stress. The model, fig. 10, shows at once that to obtain the full resistance from the material employed to resist tension or compression consistently with a given depth, it must all be placed in two horizontal plates at the extreme top and bottom of the beam; moreover, if $\frac{1}{2}$S is the sectional area of the top or bottom member, and f is the stress it will bear (assumed equal for tension and compression), the maximum moment which the elastic forces can exert is the sum of the moments due to the top and bottom members about the axis or tongue, *i.e.*, $2(\frac{1}{2}f\mathrm{S} \times \frac{1}{2}d)$ or $\frac{1}{2}f\mathrm{S}d$; or dismissing the idea of an axis through the centre of gravity of the section, we may (to refer again to the illustrative model) take the moment of the elastic forces round the line running through the top row of india-rubber pieces looked upon as a fulcrum, and then the moment of the bottom row will be as before $\frac{1}{2}f_t\mathrm{S}d$, and will be the *whole moment* of the elastic forces. Similarly, by taking the bottom row as the fulcrum, we see that the moment due to the top row will be the same. This moment calculated in either way is the *whole* moment of the elastic forces. This latter mode of considering the moment in the simple case of an I or hollow rectangle, enables us to find the value of the moment of elastic forces for those cases in which the ultimate strength may not be the same for tension and compression. Thus, assume that the strength of wrought iron to resist tension, or f_t, is 25 tons, and its strength to resist compression, or f_c, is 20 tons, then calling S_t the area of the bottom member at the section considered, and S_c the area of the top member, we have for μ the two values 25 $\mathrm{S}_t d$ and 20 $\mathrm{S}_c d$; whichever happens to be the smaller will represent the available value of the moment of the elastic forces. Consequently we must, in order not to waste material, design the beam so that the ratio of the material in the upper member to that in the lower member shall be 5 to 4. That this ratio ought to be adopted is evident from the fact that the strength of the beam will be limited by that of the weaker member. No structure is perfectly designed unless it will when overstrained give way simultaneously in every part. The foregoing theory, the soundness of which is borne out by experiment, tacitly assumes that, although the strength differs, the modulus of elasticity is constant for tension and compression. For example, if the flanges are made with sections bearing to one another the proportion of 4 to 5, the neutral axis (neglecting the web) will, assuming E constant, be at a distance from the top or larger flange equal to $\frac{4}{9}$ths of the depth; then the intensity of stress varying directly as the distance from the neutral axis will for the two flanges be in the desired ratio of 4 to 5. Thus we see not only that the I, or hollow rectangle, has the advantage of being the best form for a girder, but that it allows us easily to arrange the material to the best advantage, even when its strengths to resist tension and compression are dissimilar. Values of the modulus of rupture given above are therefore not to be applied to this design of beam, but the values of f_t and f_c are to be taken from Tables I. and II. In the case of cast-iron the member under tension is made with six times the cross section of the member under compression, the reason being the same as that for making the ratio of the upper and lower members of the wrought iron beam 5 to 4. When a cast-iron beam is thus designed, the moment which any section can exert will for a given depth be proportional to the area S_t of the lower flange. Professor Hodgkinson verified this theory experimentally, and found that the ultimate value of the moment due to elastic forces expressed in lbs and inches for beams thus designed was $\mu = 16500\,\mathrm{S}_t d$. The value of the constant agrees closely with the tensile strength of cast-iron. The experiments by Professor Hodgkinson are therefore consistent with the assumption, that although the strength of cast-iron is very different in resisting tension and compressionn, evertheless the modulus of

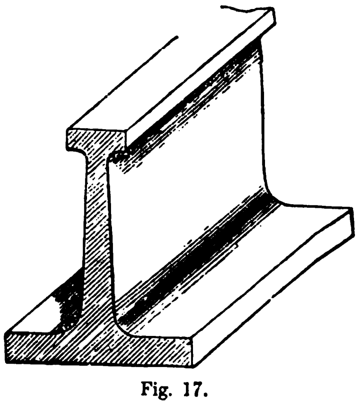

Fig. 17.

elasticity is equal under the two kinds of stress. Fig. 17 shows the cross section usually adopted for cast-iron girders.

§ 18. *Neutral Axis.*—The line ZZ_1, fig. 14, perpendicular to the plane in which a beam is bent, and passing through the centre of gravity of any given cross section, is called the *neutral axis* of the beam at that point. The surface containing all the neutral axes is the *neutral surface*. Practical engineers sometimes apply the term neutral axis to the longitudinal line showing the neutral surface on a side elevation, but in this article, as in Rankine's works, the words will be used as defined above.

When the assumption is made that the modulus of elasticity is the same for any given material whether under tension or compression, notwithstanding any difference in the ultimate strength to resist tension or compression, then it follows, as has been shown, that the neutral surface of a bent beam will separate it into two parts, one of which is compressed while the other is extended. The neutral axis in any cross section then contains the only part of the material which is neither extended nor compressed.

If, however, the average value of E for stresses varying between zero and the maximum intensity of compression to which the beam is subjected be different from the average value between zero and the maximum intensity of tension, then the neutral axis as above defined will not be the *unstrained* axis; the neutral axis is determined as soon as the cross section of the beam is known, being independent of the material used; the unstrained axis may differ in beams of the same cross section but made of different materials; for if the average of E be greater say for compression than for tension, this will raise the unstrained axis above the neutral axis. It is not improbable that the position of the unstrained axis may vary in the same beam with loads similarly distributed, but of different magnitude, and also with different distributions of load. Until experiments shall have accurately determined the relation of E to the intensity of stress we have no means of determining accurately the position of the unstrained axis. Even when E is constant the neutral axis, as above defined, will not always in practice correspond with the unstrained axis; for instance, in a beam which was not only bent across, but also compressed endways, the unstrained surface would no longer contain the neutral axis as above defined. The unstrained surface might be near one edge of the beam, or, indeed, if the general compression were large and the bending small, the whole beam might be under compression, so that no part was unstrained. By restricting the use of the words neutral axis to the above definition, and using the words unstrained axis, or unstrained surface, for the second idea all ambiguity will be avoided.

The actual position of the unstrained axis in any beam of any material subject to a bending moment depends on the relative values of the modulus of elasticity for the material under all stresses positive and negative, great and small; but as the simple hypothesis of a constant modulus is borne out by experiment in the most important materials, it is unnecessary to pursue this subject further.

§ 19. *Shearing Stress.*—The theory hitherto given shows the relation between longitudinal stresses (such as are resisted by the india-rubber in the model) and the load on the girder; but in designing a girder we have also to provide for the shearing stress or transverse force tending at each imaginary cross section to make the more heavily loaded of the two parts into which it divides the bridge slide down past the other. This shearing stress was resisted by the tongues of the model. The total shearing stress at any section is the sum of all the vertical forces acting on the beam on one side of the section. The shearing stress at any section will be called positive if the sum of the external forces on the right hand part of the beam tends to lift that portion up. Diagrams may be conveniently used to show shearing stresses, and, as in the case of bending moments, the shearing stress at any section due to two or more loads is simply the algebraic sum of the shearing stresses due to each load.

Example 1. Load W at centre between supports (fig. 18); weight of beam neglected. The shearing stress is equal to $\frac{1}{2}$W all along the beam, being the reaction at one pier; the stress is positive to the right of the load, negative to the left.

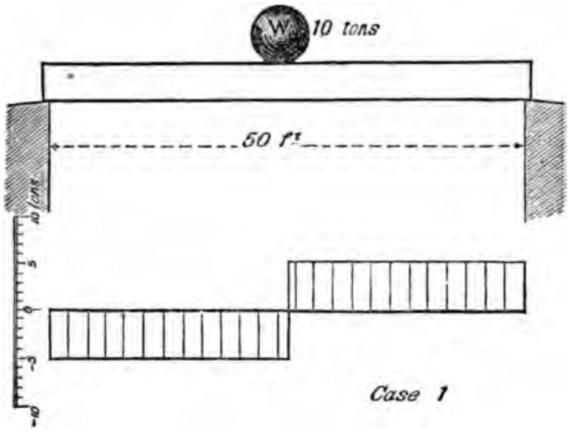

Fig. 18.

Example 2. Uniformly distributed load w per foot run (fig. 18*a*). The shearing stress is $\frac{1}{2}w$L at the points of support, and zero at the centre of the span; at any section distant c from the centre of the beam it is wc.

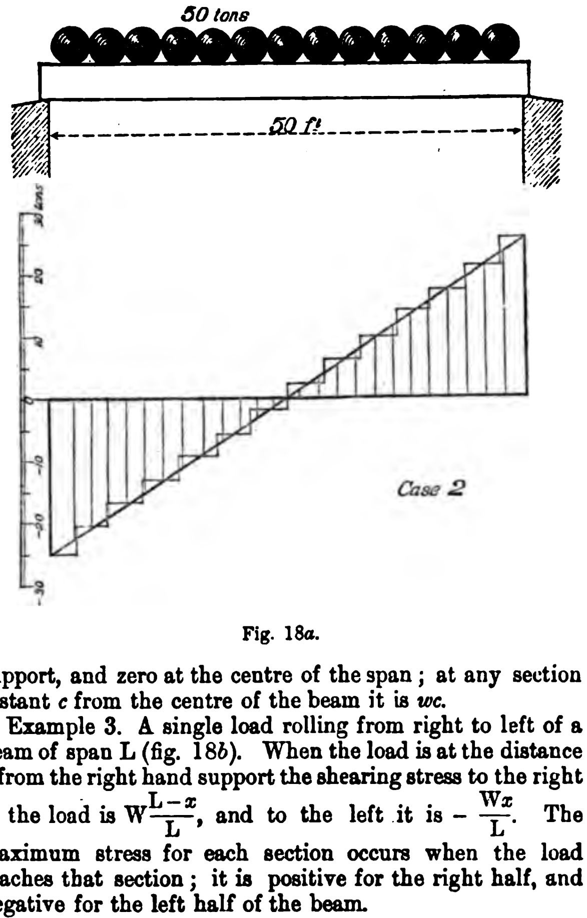

Fig. 18*a*.

Example 3. A single load rolling from right to left of a beam of span L (fig. 18*b*). When the load is at the distance x from the right hand support the shearing stress to the right of the load is $W\frac{L-x}{L}$, and to the left it is $-\frac{Wx}{L}$. The maximum stress for each section occurs when the load reaches that section; it is positive for the right half, and negative for the left half of the beam.

Example 4. Uniform advancing load of w per unit of length (fig. 18*c*). When the load covers a length x measured from the right hand pier, the shearing stress at all points beyond x towards the left is $-\frac{wx^2}{2L}$; when x is greater than $\frac{1}{2}$L the above expression gives the maximum stress which can occur on that section with any distribution of the given uniform load. Thus the maximum shearing stress at the left end occurs when the whole bridge is loaded, and is $-\frac{1}{2}w$L; the maximum stress at the centre occurs when the bridge is half loaded, and is equal to $-\frac{1}{8}w$L. The maximum shearing stresses on the other half of the beam occur when the load comes on from the left side, and covers more than half of the beam; these stresses are equal in amount to the stresses in the left half, but are positive in sign.

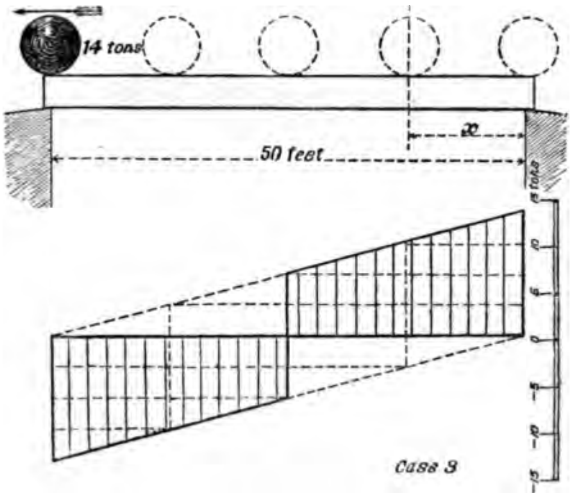

Fig. 18*b*

The scales in figures 18, 18*a*, correspond to the shearing stresses in examples 1 and 2 for a span of 50 feet and loads of 10 tons and 1 ton per foot run respectively. The scale in fig. 18*b* corresponds to the shearing stresses in example 3 with a single passing load of 14 tons. The scale in fig. 18*c*, example 4, gives the maximum shearing stress which the advancing load of one ton per foot run can produce at each section. As a train leaves a bridge it produces the same shearing stresses as when it comes on to the bridge from the opposite end, the same portions being similarly loaded. The maximum shearing stress due to a passing load of this kind changes its sign at the centre of the span, as appears by diagram 18*c*.

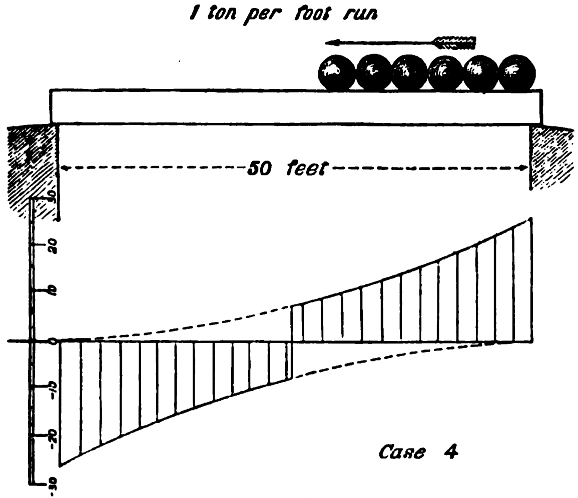

Fig. 18*c*.

In I girders with solid vertical webs the shearing stress is practically all borne by the web unassisted by the top or bottom members, it being clear that these would fold down at the sides under a small fraction of the total stress. Sufficient material is therefore employed in the web or upright plates to reduce the intensity of the stress to the desired amount. The shearing stress may, however, when the web is a thin iron plate, cause failure by crumpling the web, or causing it to buckle, instead of by shearing it across. This tendency is prevented by stiffening the web with angle or T irons rivetted to the sides. Mathematical analysis has not yet been very successfully applied to the determination of the amount of stiffening required; experience has given a sufficient number of examples to guide the practical designer. In cast-iron girders the web is generally much in excess of the strength required to resist shearing.

§ 20. *Factor of Safety.*—In designing a girder the load which it will have to carry is multiplied by a number called the factor of safety, varying from 3 to 6, and the girder is so designed that it shall not yield at any point with less than the load thus multiplied. If, for instance, the girder is practically to carry 1 ton per foot run, it is designed so that at no place shall it *break or yield injuriously* with less than say 5 tons per foot run. The multiplier is called the *factor of safety*. The factor of safety is required to allow for imperfections in the material as compared with picked specimens, for the wear and tear by which the strength of a structure is gradually reduced, for unforeseen loads, for jars and vibrations, for imperfection of theory, and for the sake of obtaining stiffness. This last property might be the subject of calculation, and in some cases must be separately examined. The particular factor employed depends on the judgment of the engineer. A larger factor of safety is required for a passing or moving load than for a permanent load, there being a greater uncertainty as to the stress which may be caused by vibrations or impulses due to what is sometimes called a live load. Moreover, the mere presence of a large permanent load tends by its inertia to diminish the dangerous effect of the impulses or stresses due to the passing load, so that the factor of safety should be chosen with reference to the ratio $\frac{\text{max. passing load}}{\text{max. permanent load}}$ being larger as this ratio increases. Rankine recommends that the factor of safety should for the moving load be double that employed for the permanent load. Sometimes the factor is more conveniently employed as a divisor to deduce the safe stress f_1 from the ultimate strength f of the material, rather than as a multiplier for the load. Thus the same number of square inches will be obtained in the tension member of a wrought iron girder to bear 1 ton per foot run, whether we use in the calculations 25 tons as the value of f, the ultimate strength of the material, and a load w of 5 tons per foot run, or if we use 5 tons as f_1, the safe stress on the material, and a load w of 1 ton per foot run; in short, if we call the factor of safety K, we may in equation 5, § 14, use $\text{KM} = \frac{2f\text{I}}{d}$, or making $f_1 = \frac{f}{\text{K}}$, we may write

$$1. \quad . \quad . \quad . \quad \text{M} = \frac{2f_1\text{I}}{d}.$$

The same remark applies, of course, to equation 6, § 14.

§ 21. *Weight of Girders and Roadway.*—When two girders are employed for each line of way on a railway, the weight of the iron girders per foot run will, with the usual proportions, probably lie between 0·0017L tons and 0·0025L tons, L being the span in feet. It follows from the theory given above, that for similar beams the quantity of material in the whole girder will be proportional to the square of the length, and, therefore, the quantity per foot run will be proportional to the simple length. The constants given above are derived from practice. The weight of girders for a common road, if placed from 7 to 8 feet apart, will be nearly the same as for railway girders of the same span. The weight of a cast-iron railway girder (two girders per way) will be about 0·005L tons per foot run. The weight of the *roadway* in a railway bridge will probably be from 0·14 to 0·22 tons per girder, or double this for each line. For a turnpike road with metalling the weight will much exceed this, and should in each case be computed.

§ 22. *Design of a Girder.*—(1.) From the span and load to be carried the engineer will determine the material and form to be employed. Cast-iron may in some districts be the cheapest material for girders under 30 feet span. Wrought iron I girders are very generally employed for spans of from 30 feet to 100 feet; beyond that span lattice or framed girders are more usually employed. For extreme spans exceeding, say, 300 feet, a hollow rectangle or tubular bridge may be used, carrying the road on its top or inside the tube. The depth of the cross section is limited by the consideration that the web must be sufficiently stiff not to buckle; but for this consideration the deeper a girder could be made the better. In practice the depth is made from $\frac{1}{8}$th to $\frac{1}{15}$th of the span. The engineer will also determine whether he will keep the depth of the girder constant throughout or diminish the depth at the ends. It is impossible to graduate the material so as to give absolutely uniform strength at all sections, but by diminishing the depth towards the ends, some material may be saved without attenuating the top and bottom members to such an extent as to be inconvenient. When the general character of the design has thus been settled, the engineer will compute the probable weight of the girders and roadway or total permanent load; he will next determine the passing load for which he intends to provide.

(2.) The value of M, the bending moment, must next be computed for a sufficient number of cross sections of the beam, and for various distributions of load. For a small cast-iron girder of uniform cross section a single value of M will be sufficient, computed for the section at the centre when the girder is wholly covered with the greatest uniform load and also supports the greatest single load at the centre of the span. When, as in larger girders, the design is intended to give a structure of approximately equal strength throughout, the maximum value of M should be found for eight or ten sections; this maximum value will be that obtained n the bridge is wholly loaded with its maximum uni-
ι load and has the maximum single load resting just the section in question.

.) The maximum shearing stress must next be calculated ach of the above sections. The designer will bear in . that the maximum stress occurs at the points of ɔrt, and that at the centre it is greatest when the e is half covered with the passing load.

) The engineer can now compute the number of square s S_t and S_c required at each section in the upper and members consistently with the factor of safety he s to employ; this he obtains from the expressions—

$$. \quad . \quad . \quad . \quad . \quad . \quad . \qquad S_t = \frac{M}{f_{t_1} d},$$

$$. \quad . \quad . \quad . \quad . \quad . \qquad S_c = \frac{M}{f_{c_1} d}.$$

ıere assumed that the best and strongest form of s employed, but if a mere square or circular beam used, the cross section will be obtained by equating ıes of M and μ, using a safe modulus of rupture f_1. The web will next be designed by giving it such a s as will, with the depth already fixed, supply the number of square inches required to reduce the stress per square inch to the *safe* or *proof* shearing stress, say 4 or 5 tons on wrought iron. When the web is a thin wrought iron plate it must be stiffened with ⊥ or angle irons. In a cast-iron girder the web must have at least the number of square inches required by the shearing stress, but the exigencies of the foundry generally require a design resulting in a great excess of strength in this part of the beam, except in beams which are tapered towards the ends, as in fig. 19. With these beams care must be taken that the taper is not carried to excess so as to leave insufficient metal to resist the shearing stress at M and N.

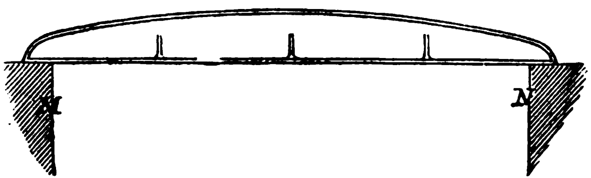

Fig. 19.

§ 23. *Practical Details.*—The designer must be practically acquainted with the forms in which his materials can be best procured. He must know the sizes in which iron or steel plates can be produced, and the forms best adapted for castings. Thus, in cast-iron beams the thickness of the web is at the bottom made equal to the thickness of the lower flange, and at the top to the thickness of the upper flange, in order to avoid permanent internal strains, which would result from unequal rates of cooling after being cast, if sudden changes of thickness in the metal were allowed. The engineer must also be familiar with the methods adopted of joining the several parts, as with the rivetting of wrought iron, the bolting together of large castings, the jointing of wood-work. He should also be acquainted with the various methods in which roadways are constructed and supported on existing bridges, and the manner in which the girders are braced one to another, so as to prevent vibration and lateral deflection due to the pressure of the wind. The examples of bridges described hereafter will give some information on these points. In long girders provision must be made by rollers, sliding plates, or suspension links for the expansion and contraction due to changes of temperature. The range in Great Britain may be taken as about 45° C. If the ends of the girder could be firmly secured at a constant distance apart this change of temperature would produce a stress of about 6 tons per square inch in wrought iron, and 3 tons per square inch in cast-iron. The result in practice would be that any attempted fastening of stone or iron work would be torn loose.

§ 24. *Deflection.*—When a bridge has been erected its deflection at the centre under a known passing load is generally observed with the object of ascertaining whether the work has been properly done, for it is assumed that any defective material or bad jointing would increase the deflection beyond that calculated on the assumption of sound material and perfect workmanship. Sometimes the practical test applied is a rough one, a certain fraction of an inch being allowed per foot of span as a safe deflection. If an inspector of bridges, having authority, chooses to limit the deflection to a constant fraction of the span, the ratio of the depth to the span must be made sufficiently great to give the desired stiffness and maintained constant for all spans; equation 5 below shows that when p_1 is kept constant and d is a given fraction of L, the deflection v will be proportional to the span. For the proof or maximum possible load, Rankine gives as the result of practice a value for the deflection of from $\frac{1}{200}$L to $\frac{1}{600}$L; but one foot deflection in a span of 200 feet would certainly be exces-

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sive. A depth equal to or greater than $\frac{1}{12}$th of the span is certain to give sufficient stiffness, and the usual method is to assume the depth and then to observe whether experiment gives a deflection agreeing with that found by calculation. The calculation is made by finding the radius of curvature of the beam at a series of sections, and then determining the curve assumed by the whole beam either by integration or by an approximate graphic method. When the curve is known the deflection or versed sine is found either from the equations of the curve, or by actual measurement on the diagram to be presently described. Let R be the radius of curvature of the neutral surface of a beam at a given section, under a load producing a maximum stress p_1 at the outer elements of the section at a distance y_1 from the neutral surface. Consider a short length x of the beam fig. 20. Before deflection the length of the outer element is equal to x, but after deflection (if we consider the upper member) the length of the outer element will be shortened to x_1, while the length of the element in the neutral surface remains equal to x. By similar triangles we have $x : x_1 = \text{R} : \text{R} - y_1$, or $\text{R} = \frac{y_1 x}{x - x_1}$, but $x - x_1$ is the extent to which the most compressed element is shortened, and by the definition of the modulus of elasticity we have $\text{E} = p_1 \frac{x}{x - x_1}$, hence

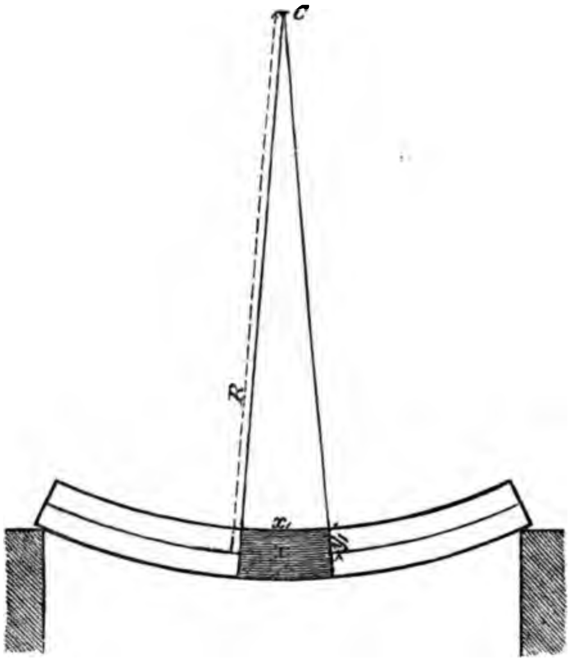


Fig. 20.

$$1. \;.\;.\;.\;.\;.\;.\quad \text{R} = \frac{\text{E}y_1}{p_1},$$

from which the radius of curvature at any section can be obtained in terms of E, y_1, and p_1, all known quantities with a given cross section, material, and load. Another form of the same expression, which is sometimes more convenient, is obtained by remembering that $\text{M}_1 = \frac{p_1 \text{I}}{y_1}$, hence

$$2. \;.\;.\;.\;.\;.\;.\quad \text{R} = \frac{\text{EI}}{\text{M}_1},$$

where M is the bending moment which will produce p_1. If p_1 is made equal to f_1 the maximum safe stress on the material, equation 1 or 2 gives the minimum safe radius of curvature.

We will first consider the special case in which the beam under consideration is of *equal depth and uniform strength* at all cross sections, and which we will call a beam of class 1; these conditions can only be fulfilled by any one beam for a given constant distribution of load; the beam of uniform strength for a load at the centre, for instance, will clearly not be the beam of uniform strength for a uniformly distributed load. In beams of class 1 for a given load, both y_1 and p_1 are constant, and therefore R, by equation 1, will also be constant throughout the whole length of the beam, or, in other words, the beam will bend into a circular arc. The approximate expression for the versed sine of an arc having a chord L and radius R will therefore give the deflection, and we have

$$3. \;.\;.\;.\;.\;.\;.\;.\quad v = \frac{\text{L}^2}{8\text{R}},$$

and employing for R the value given by equation 1, we have

$$4. \;.\;.\;.\;.\;.\;.\;.\quad v = \frac{\text{L}^2 p_1}{8\text{E}y_1},$$

and if $y_1 = \frac{1}{2}d$, we have

$$5. \;.\;.\;.\;.\;.\;.\;.\quad v = \frac{\text{L}^2 p_1}{4\text{E}d}.$$

If for p_1 we substitute f_1, the maximum safe stress for the given materials, equation 4 or 5 will give the maximum safe deflection, which may be called v_1; we observe, then, that the safe deflection for a beam of this class will be proportional to the square of the span, and inversely proportional to the depth of the beam.

In beams of class 1, the deflections which different loads produce will be simply proportional to the values of p_1 produced by those loads; thus, for a given distribution of load, the deflection will be simply proportional to the load; if we change the distribution of the load, keeping the total load constant, the same rule will give the solution; for instance, since a load uniformly distributed produces only half the stress p_1 which would be produced by the same load at the centre of a beam of the same span and cross section, we see that the uniformly distributed load will produce only half the deflection that would be produced by the same load at the centre of a beam of the same span and same *section at the centre* (both beams belonging to class 1). It would not be correct to say that a load uniformly distributed would produce half the deflection produced by the same load at the centre of the *same beam*, because the same beam cannot be uniformly strong throughout its length for two different distributions of load.

We may compare the deflections produced by the same load on various beams of similar cross section as follows:—By equation 4, § 14, we see that for a given moment M, p_1 is proportional to $\frac{d}{\text{I}}$; moreover, for equal loads, the moment M at any cross section similarly placed will be proportional to L; hence we may write

$$p_1 \propto \frac{\text{L}d}{\text{I}} \propto \frac{\text{L}}{bd^2}.$$

Substituting this expression for p_1 in equation 5, we have

$$6. \;.\;.\;.\;.\;.\;.\;.\quad v \propto \frac{\text{L}^3}{bd^3},$$

an equation which expresses the fact that, for beams of class 1, the deflection produced by a given total load similarly applied will be proportional to the cube of the length, and inversely proportional to the breadth and to the cube of the depth. (In I girders the expression breadth must be understood as proportional to S, the section of the flange.)

Passing from beams of class 1 to beams in general, the fundamental difference to be observed will be that the curve assumed by the neutral surface will not be that of a circular arc, so that equation 1 is no longer true. Where, however, the law according to which R varies can be stated algebraically, integration will give the value of v.

Combining equations 2 and 3 for class 1, we have $v=\frac{1}{8}\frac{M_1L^2}{EI}$, and as $M_1 \propto LW$, we have $v \propto \frac{WL^3}{EI}$. Now this proportional equation can be shown to hold good for beams of uniform cross section and uniform depth, which may be called beams of class 2, also for beams of uniform cross section and uniform breadth, which may be called beams of class 3, hence for the three classes of beams we may write

$$7. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad v=n\frac{WL^3}{IE},$$

where n will have different values in the three classes and for each distribution of load.

Similarly it can be proved, that where beams are so designed that the loads produce the same maximum value p_1 of the stress on the outer elements, we have the deflection proportional to $\frac{p_1L^2}{Ey_1}$,—equation 4 being one case of the general law; we may therefore write

$$8. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad v=n_1\frac{p_1L^2}{Ey_1},$$

where n_1 is a constant differing for each class of beam and each distribution of load.

Table X. gives the values of n and n_1 for the three classes of beam, and for two distributions of load. The value of p_1 in the case of a beam of uniform cross section applies to the stress at the centre; the stresses elsewhere in that beam will be less.

TABLE X.—*Values of* n *and* n_1.

Description of Beam.	W at Centre.		W uniformly distributed.	
	n	n_1	n	n_1
Uniform strength and depth ..	$\frac{1}{32}$	$\frac{1}{8}$	$\frac{1}{64}$	$\frac{1}{8}$
Uniform strength and breadth	$\frac{1}{24}$	$\frac{1}{6}$	0·0178	0·1427
Uniform cross section..........	$\frac{1}{48}$	$\frac{1}{12}$	$\frac{5}{384}$	$\frac{5}{48}$

In any actual bridge girder the deflection should lie between the value calculated for the beam of uniform strength and that calculated for the beam of uniform cross section.

§ 25. *Graphic Method of finding Deflection.*—Divide the span into any convenient number n of equal parts of length l, so that $nl = L$; compute the radii of curvature R_1, R_2, R_3 for the several sections. Let measurements along the beam be represented according to any convenient scale, so that calling L_1 and l_1 the lengths to be drawn on paper, we have $L = aL_1$; now let r_1, r_2, r_3 be a series of radii such that $r_1=\frac{R}{ab}$, $r_2=\frac{R_2}{ab}$, &c., where b is any convenient constant chosen of such magnitude as will allow arcs with the radii r_1, r_2, &c., to be drawn with the means at the draughtsman's disposal. Draw a curve as shown in fig. 21 with arcs of the length l_1, l_2, l_3, &c., and with the radii r_1, r_2, &c. (note, for a length $\frac{1}{2}l_1$ at each end the radius will be infinite, and the curve must end with a straight line tangent to the last arc), then let v be the measured deflection of this curve from the straight line, and V the actual deflection of the bridge; we have $V = \frac{av}{b}$, approximately. This method distorts the curve, so that vertical ordinates of the curve are drawn to a scale b times greater than that of the horizontal ordinates. Thus if the horizontal scale be one-tenth of an inch to the foot, $a = 120$, and a beam 100 feet in length would be drawn equal to 10 inches; then if the true radius at the centre were 10,000 feet, this radius, if the curve were undistorted, would be on paper 1000 inches, but making $b = 50$ we can draw the curve with a radius of 20 inches. If we now measure the versed sine of an arc drawn with a length 10 inches and a radius 20 inches, we shall approximately find it equal to 0·64 inches, hence $V = \frac{120 \times \cdot 64}{50} = 1{\cdot}54$ inches. The vertical distortion of the curve must not be so great that there is any very sensible difference between the length of the arc and its chord. This can be regulated by altering the value of b. In fig. 21 distortion is carried much too far; this figure is merely used as an illustration, and is not to be taken as an example.

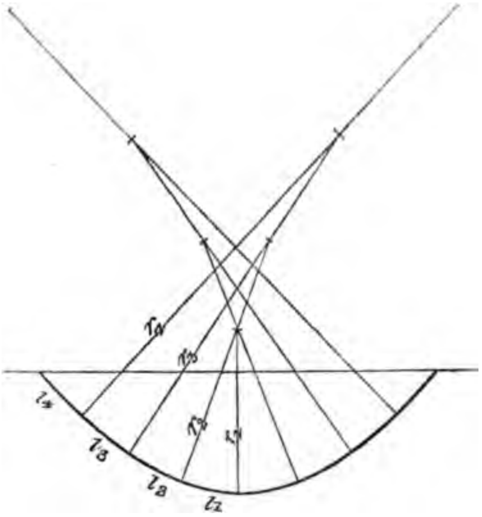


Fig. 21.

§ 26. When a girder has more than two supports it is called a *continuous girder*. The distribution of the stresses in a continuous girder differs very materially from that in a simple girder, as will be at once apparent by the inspection of fig. 22, which shows the way in which a continuous girder of two spans and two simple girders bend when employed to carry equal weights across equal openings. The continuous girder when both spans are loaded is bent upwards at C over the centre pier; in other words, the bending moment at this and neighbouring points is negative. The direction of the flexure changes at certain sections, as at A and A_1, *i.e.*, the bending moment is positive on one side of these sections, negative on the other side; and at the section where the direction of flexure changes the bending moment is *nil*. Again, when only one of two simple girders is loaded, the girder over the second span is not bent in either direction, but with the continuous girder there may be a negative bending moment produced throughout the whole unloaded span as shown in fig 23. Con-

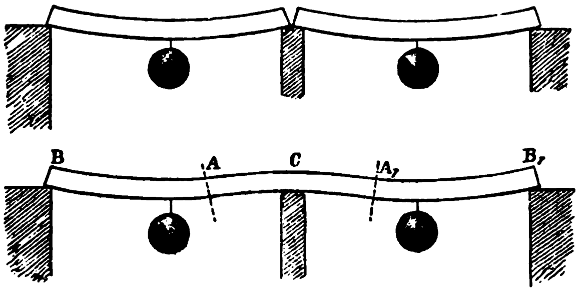


Fig. 22.

tinuous girders require less material for the same depth, span, and permanent load than simple girders; but the difference is hardly worth consideration, except in large spans where the weight of the structure is great as compared with the weight of the passing load. The necessity of allowing the girder to expand and contract freely with changes of temperature limits the general use of large continuous girders to two spans.

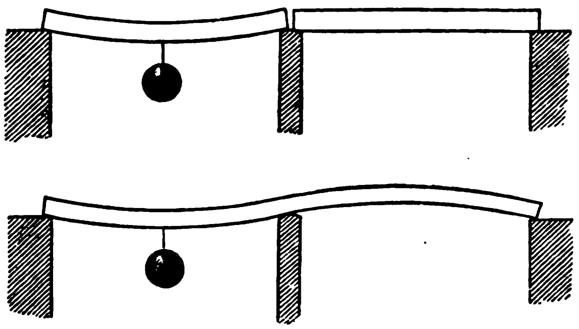


Fig. 23.

Consider a continuous girder having an indefinite number of supports equally spaced; let the girder be of constant depth, uniformly loaded, and so proportioned as to be of uniform strength at all sections. The method of designing such a girder has not yet been shown, but it may be admitted that the design is possible. Then, as we have already seen (§ 24), the curvature assumed by the girder will everywhere be constant, *i.e.*, the curves will be circular arcs of constant radius. The points of inversion of flexure A and B (fig. 24) must therefore in this case be at a distance from C and C_{11}, equal to one quarter of the span; for $AA_1 = AB = A_1B_1$ &c., and C bisects AA_1. At B and B_1 there is no moment of flexure. The girder might here be cut through, and if the ends B and B_1 were pinned on to the rest of the girder by pins capable of bearing the shearing stress, nothing would be changed in the curvature of the girder nor in the distribution of stress. Quite similarly, we may suppose the ends B and B_1 of the portion $BACA_1B_1$ to rest directly on supports or piers introduced for the purpose, the rest of the imaginary girder being wholly removed. We shall then have (fig. 25) the curve assumed by a continuous girder of two spans of constant depth and uniform strength. The points AA_1 of inversion of flexure will be at a distance CA from the middle pier equal to one-third of the span. The top and bottom members at A and A_1 might vanish (if only one distribution of load were to be carried), for at this point there is only a shearing stress and no bending moment. The girder is, as it were, made up of three girders: BA and A_1B_1 hang on to AA_1, which is supported in the middle; half the weight of AB is borne by the pier B; half the weight of A_1B_1 is borne by the pier B_1; the rest of the weight between B and B_1 is borne by the middle pier. Thus let L be the length of one span in feet, w the load per foot run, P the load borne by each of the end piers, and P_c the load borne by the centre pier, then we have—

1. . . . . $P = \frac{1}{3}wL$,
2. . . . . $P_c = \frac{4}{3}wL$.

The curve of bending moments can now be calculated for each girder precisely as for a simple girder; the moment at any section is equal to the sum of the moments of all the external forces on one side of the section; the beam between A and B will be subject to bending moments equal to those produced by a uniform load on a simple girder if the span BA be similarly loaded. Between A and A_1 the moments will be negative, *i.e.*, the left-handed moment produced by the downward action of all the weights to the left will exceed the right-handed moment produced by the upward reaction of the pier at B (or of the two piers B and C if the section considered lies to the right of C). The full black line in fig. 25*a* shows a curve of bending moments for this case.

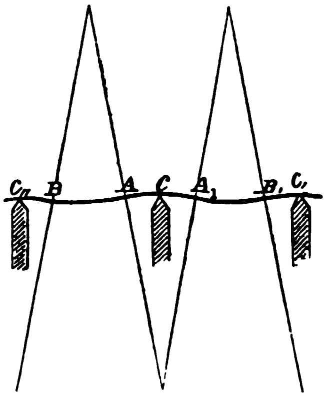


Fig. 24.

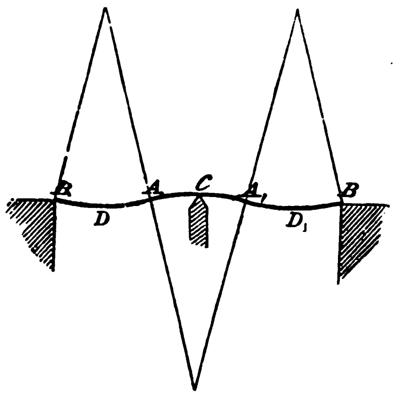


Fig. 25.

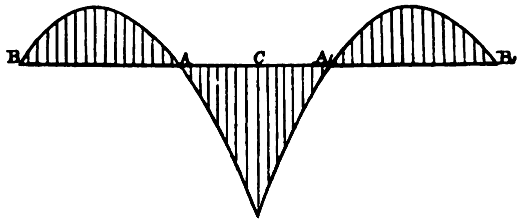


Fig. 25*a*.

The maximum moment is negative and occurs over the centre pier; let it be called M_c. The maximum positive moment occurs at a distance from B equal to one-third of the span; let this moment be M_d. Then we have—

3. . . . . . . $M_c = -\frac{1}{6}wL^2$,
4. . . . . . . $M_d = \frac{1}{18}wL^2$.

The shearing stresses F_b, F_d, F_a, F_c, at the points B D, A, and C, are

5. . . . . . . $F_b = F_a = \frac{1}{3}wL$,
6. . . . . . . $F_d = 0$,
7. . . . . . . $F_c = \frac{2}{3}wL$.

Fig. 26 gives a diagram of the shearing stresses for two spans of 60 feet, with a uniform load of 1 ton per foot run.

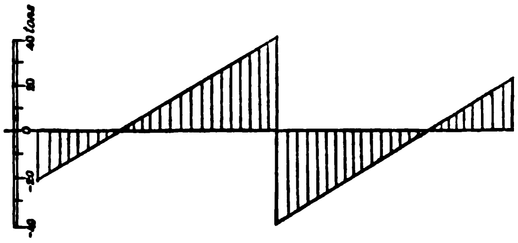


Fig. 26.

When the beam is of uniform depth and uniform cross section, the curves in which it deflects are no longer circular; the defect of strength over the centre pier has the effect of increasing the curvature over it, and shortening the distance AA_1; analysis shows that in this case the point of inversion of flexure will be at a distance from the centre pier equal to ·2683L; then the length of the part of the beam subject to a positive bending moment will be ·7317L, instead of ·66L as when the beam was of uniform strength; the load on each of the end piers will be ·36585wL; the load on the centre pier 1·2685wL; and we have—

8. . . . . . $M_c = \cdot 1341 wL^2$,
9. . . . . . $M_d = \cdot 0669 wL^2$,
10. . . . . . $F_b = F_a = \cdot 36585 wL$,
11. . . . . . $F_d = 0$,
12. . . . . . $F_c = 1 \cdot 2683 wL$

In any actual bridge uniformly loaded the values of the moments and shearing stresses will be intermediate between those given for a beam of uniform strength and those for a beam of uniform cross section. It must be observed that the above theory assumes that the girder is unstrained when being built as a whole; if, as is often the case, the separate spans are separately built and lifted into position, and then joined on the piers, special provision must be made to bring the desired bending moment over the piers into existence; it is obvious that merely joining the two independent beams in the upper part of fig. 22 will not make these into a continuous girder, such as is shown in the lower figure,—to do this, besides joining the upper and lower flanges, we must pull the top flanges together over the piers and put the lower flanges under compression. This may be done practically by tilting one end, or both ends, before the junction at the centre is made, and afterwards allowing the ends to sink until the curve assumed by the girder shows that the required distribution of stress has been attained. The complete analysis of the problem of continuous girders of any number of spans equal or unequal with any number of loads has been given by Mr Heppel (*Proc. R. S.*, 1870–71).

§ 27. *Allowance for Weight of Beam. Limiting Span.*—When the weight of the beam is a considerable and uncertain part of the whole load, it can be allowed for as follows. Design a beam of the desired depth and span, fit to carry a total load equal to the external or passing load W_1; calculate the weight of this beam and call it B_1; the beam so designed will really be fit to carry an external load $W_1 - B_1$. Let b_1 be the area of any cross section of this beam; let b be the area of cross section required at the same point for the beam of weight B actually necessary to carry a total load W. Then since the strength of the properly proportioned girder of constant depth and span is simply proportional to the quantity of metal employed, and therefore to the area of cross section, we have the proportion $b : b_1 = W_1 : W_1 - B_1$, or

1. . . . . $b = \frac{W_1 b_1}{W_1 - B_1}$.

The weight B is given by the expression—

2. . . . . $B = \frac{B_1 W_1}{W_1 - B_1}$.

The whole load W is given by the expression—

3. . . . . $W = \frac{W_1^2}{W_1 - B_1}$.

For any given design of beam there is a limiting length which cannot be exceeded (the beams of different spans being assumed to be *similar* in the geometrical sense). Let $L_{,}$ be the limiting length of a beam of a given design which for the span L weighs B, and carries a gross load W, then the ratio of B to W can be shown to increase in direct proportion to the length of the span until this ratio reaches unity. Hence—

$$\frac{B}{W} \cdot \frac{B_{,}}{W_{,}} = L_{,} : L_{,},$$

or when

$$\frac{B_{,}}{W_{,}} = 1,$$

4. . . . . $L = \frac{WL}{B}$.

III. Suspension Bridges.

§ 28. *Varieties of Suspension Bridges.*—A very simple form of suspension bridge has long been used in Peru and Thibet. Two ropes are hung side by side across the gorge to be passed, a rude platform is laid on the ropes, and the dip of these is sufficiently small to allow the bridge to be crossed by men or beasts passing down from the one side to the centre and up to the opposite bank. The modern suspension bridge consists of two or more *chains*, from which a level *platform* is hung by suspension *rods*. The chains may in some cases be secured directly to the sides of the chasm to be crossed, but the configuration of the ground seldom allows this to be done. The chains, therefore, usually pass over *piers*, as in fig. 2, Plate XIX., and are led down on either side to an *anchorage* at a considerable distance from the piers. The chains between the piers and the anchorage are generally used to support part of the platform. The chains where they pass over the piers rest on *saddles*, which are made of two different types. One construction, shown in fig. 27, allows the chain to slip backwards and forwards over it with comparatively little friction, so that the stress on the rope may be taken as equal on both sides of the saddle. In the second type, as shown in fig. 28, the chain is secured to the saddle, which, however, is free to move horizontally on the top of the pier. With the first form of saddle the resultant pressure on the pier will not be vertical unless the chain leaves the pier with an equal inclination on each side, and even when the bridge is designed with an equal slope of chain on both sides of the pier, a change in the distribution of weight due to any passing load will cause some departure from the equal slope of the chains, and therefore from the truly vertical pressure on the piers. This departure is easily allowed for in the design of the pier. The friction on the saddle renders the

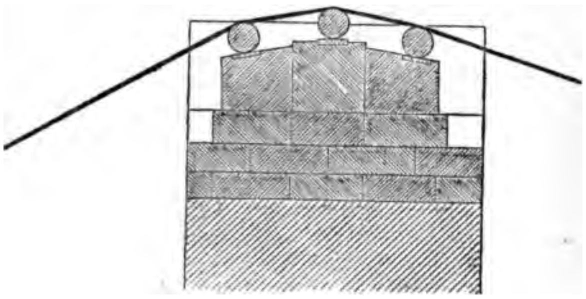


Fig. 27.

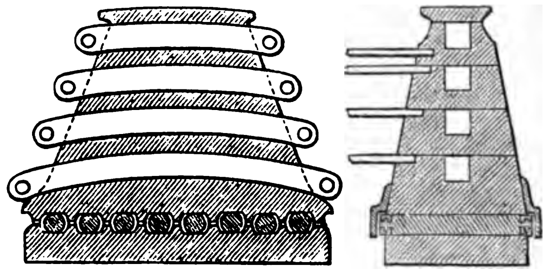


Fig. 28.

assumption of equal stresses on each side slightly incorrect, and with this type of saddle care must be taken to provide against the wear produced by the motion of the chain. With the second type the use of rollers under the solid saddle leaves the motion of the saddle very free; the resultant pressure on the pier is always sensibly vertical, and the chains may leave the pier at any angle, equal or unequal. The chain must in no case be rigidly attached to the pier unless the support itself is free to rock on its base, as in fig. 29, where the place of the pier is taken by cast-iron struts, working on a horizontal axis.

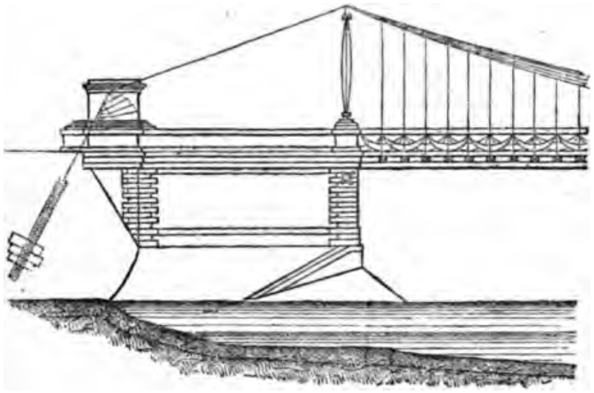


Fig. 29.

Suspension bridges are chiefly used for very large spans, because, as we shall find, they can be constructed to carry the same load with a less weight of material than a beam or girder, subject, however, to the disadvantage of flexibility or deformation under a passing load,—a disadvantage which is very serious where, as in small bridges, the passing load is a large proportion of the whole load, but which is of less importance where the chief load to be carried is the weight of the structure itself. We will first consider the usual or simple suspension bridge, as shown in fig. 2, Plate XIX., and will then pass to the various modifications introduced to remedy its defects.

§ 29. *Form of Chain with given Load.*—Let the platform be hung from the chain by equidistant vertical rods; then the load may be treated as hanging from each joint where the rods are attached, and will consist at each joint of the weight of one subdivision of the chain and of one subdivision of the platform and its load. If the position of the vertical tie-rods be assumed as definite relatively to the points of suspension, so that the assumed loads on the joints act at known distances from the points of support, a form of chain, which will remain in equilibrium (or undisturbed) under these loads, can easily be found by the following graphic method:—

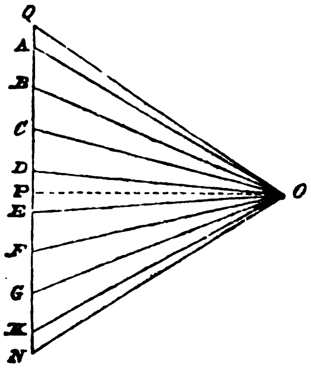


Fig. 30.

Let the vertical line QN (fig. 30) represent the whole load to be carried, and the subdivisions QA, AB, BC, CD, &c., the loads referred to each joint of the chain (QA and NK will be the portion of the load referred directly to the saddle or point of support, and will be simply half the weight of the piece of chain between the saddle and the joint pin). Let QP and NP be the weights carried by each pier,—equal if the distribution of load is symmetrical, otherwise to be determined as for a girder. Let a horizontal line, PO, represent the horizontal component of the tension to be allowed on the chain, or the whole tension on that part of the chain the tangent to which is horizontal; join O with Q, with A, B, &c.; then the lines OA, OB, &c., give the slopes of each successive link as shown in fig. 31, where the line parallel to OA in fig. 30 lies between the two spaces containing the letters O and A in fig. 31, similarly the line parallel to OB in fig. 30 is represented by the link between the two spaces lettered O and B in fig. 31, and so forth. The line in fig. 31 lying between O and Q parallel to OQ in fig. 30, represents the direction of the force on the point of support *a*, being equal and opposite to the resultant of the tension on the first link and the weight carried directly by the support. The triangle QOA (fig. 30) is the polygon of forces in equilibrium at the point of support *a* (fig. 31). The triangle OAB is the polygon of forces in equilibrium at the first joint, and similarly each component triangle of fig. 30 represents the equilibrated forces at one joint of the chain in fig. 31. This theorem is one example of the general theory of reciprocal figures, which will be treated hereafter under the general head of "Frames," § 53.

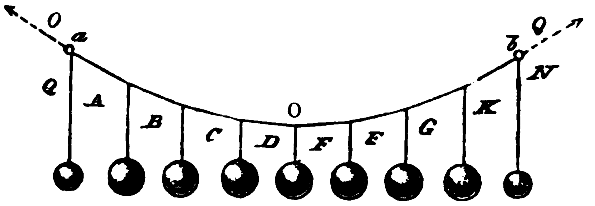


Fig. 31.

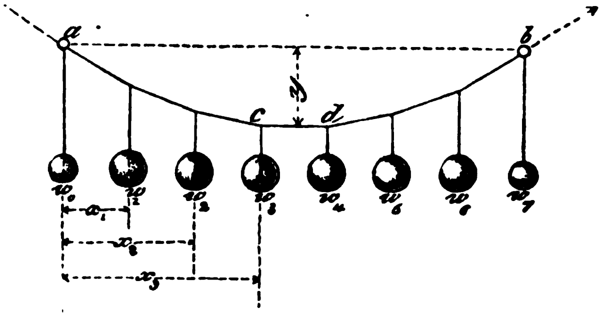


Fig. 32.

When the maximum dip is given instead of the horizontal component of the stress, it is easy to find the latter from the former by the method of moments when the point is known where the chain will be horizontal; for then, let the link *cd*, fig. 32, be horizontal; let the dip be called *y*, and let the distances of weights w_1, w_2, w_3, &c., from the point of support *a* be called x_1, x_2, x_3, &c., and let the horizontal tension represented by PO in fig. 30 be called H. Then taking moments round the point *a*, we have—

$$1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad Hy = \Sigma wx,$$

from which H can always be found.

When the length is given of each link (or portion of the chain between the joints where the platform is suspended), and consequently the length of the whole chain, the problem of determining the form assumed under any distribution of load is difficult, for the proportion of the load carried by each pier and the position of each load relatively to the piers vary when the form of the chain varies. The problem may be solved tentatively, but it is seldom attempted.

The converse problem of finding the load which will keep a chain in equilibrium when the dimensions and curve are given is perfectly easy.

From a point O, fig. 30, draw a series of lines parallel to the given links. At any convenient distance, OP, draw a vertical line cutting the lines diverging from O at the points Q, A, B, C, D, &c. The vertical loads required to keep the chain in equilibrium are proportional to the lengths QA, AB, BC, &c.

§ 30. *Relation between the Curve of Bending Moments and the Curve assumed by a Loaded Chain.*—The vertical ordinates at the joints of an equilibrated chain, measured from a horizontal axis passing through the two points of support (these being at the same level), are proportional to the bending moments for similarly chosen sections of a girder similarly loaded.

Let us consider any joint, say that at which w_3 is hanging in fig. 33. Let V be the vertical component of the resultant pull on the left hand pier, and H the horizontal component. We know that the vertical component is equal to the whole reaction at the pier when the same load is carried by a girder; then, taking moments about the joint in question, we have—

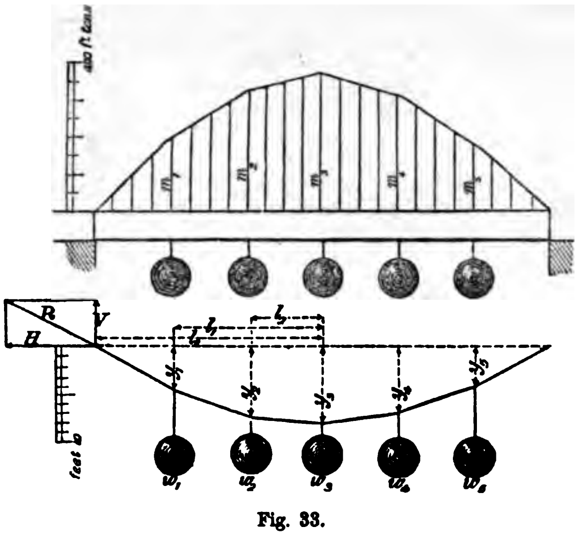


Fig. 33.

1. . . . . . . $Hy_3 = Vl_0 - w_1l_1 - w_2l_2$,

but the second member of the equation is the bending moment m_3 for the section at a distance l_0 from the pier in a girder of similar span and similarly loaded, therefore, whatever may be the value of H, the values of y_1, y_2, y_3, &c., are proportional to the bending moments. If, then, a curve of bending moments with the ordinates m_1, m_2, m_3, &c., be drawn for a given distribution of load, we can, with a pair of proportional compasses, construct any number of equilibrated curves, by making the values of y in these curves simply proportional to the values of m in the curve of bending moments, and by selection among these a curve of any required length may be found. If H be unity, the ordinates y_1, y_2, y_3, &c., are equal to the bending moments.

§ 31. *Chain Loaded uniformly along a Horizontal Line.*—If the lengths of the links be assumed indefinitely short, the chain under given simple distributions of load will take the form of comparatively simple mathematical curves known as catenaries. The true catenary is that assumed by a chain of uniform weight per unit of length, but the form generally adopted for suspension bridges is that assumed by a chain under a weight uniformly distributed relatively to a horizontal line. This curve is a parabola.

From equation 1, § 30, remembering that Σwl in this case will be equal to $\frac{wL^2}{8}$, we see that the horizontal tension H at the vertex for a span L (the points of support being at equal heights) is given by the expression—

1. . . . . $H = \frac{wL^2}{8y}$,

or, calling x the distance from the vertex to the point of support,

$$H = \frac{wx^2}{2y}.$$

The value of H is equal to the maximum tension on the bottom flange, or compression on the top flange, of a girder of equal span, equally and similarly loaded, and having a depth equal to the dip of the suspension bridge.

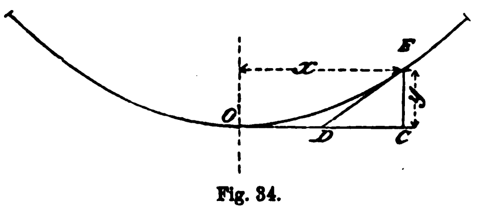


Fig. 34.

Consider any other point F of the curve, fig. 34, at a distance x from the vertex, the horizontal component of the resultant (tangent to the curve) will be unaltered; the vertical component V will be simply the sum of the loads between O and F, or wx. In the triangle FDC, let FD be tangent to the curve, FC vertical, and DC horizontal; these three sides will necessarily be proportional respectively to the resultant tension along the chain at F, the vertical force V passing through the point D, and the horizontal tension at O; hence

$$H : V = DC : FC = \frac{wx^2}{2y} : wx = \frac{x}{2} : y,$$

hence DC is the half of OC, proving the curve to be a parabola.

The value of R, the tension at any point at a distance x from the vertex, is obtained from the equation—

$$R^2 = H^2 + V^2 = \frac{w^2x^4}{4y^2} + w^2x^2,$$

or,

2 . . . . . $R = wx\sqrt{1 + \frac{x^2}{4y^2}}$

Let i be the angle between the tangent at any point having the co-ordinates x and y measured from the vertex, then

3. . . . . $\tan i = \frac{2y}{x}$.

Let the length of half the parabolic chain be called s, then

4. . . . . $s = x + \frac{2y^2}{3x}$.

The following is the approximate expression for the relation between a change Δs in the length of the half chain and the corresponding change Δy in the dip:—

$$s + \Delta s = x + \frac{2}{3x}\left\{y^2 + 2y\Delta y + (\Delta y)^2\right\} = x + \frac{2y^2}{3x} + \frac{4y\Delta y}{3x} + \frac{2\Delta y^2}{3x},$$

or, neglecting the last term,

5. . . . . $\Delta s = \frac{4}{3}\frac{y\Delta y}{x}$

and

6. . . . . $\Delta y = \frac{3}{4}\frac{x}{y}\Delta s$.

From these equations the deflection produced by any given stress on the chains or by a change of temperature can be calculated.

If the points of support are not at equal height (fig. 35) call the heights above the vertex y and y_1, and the horizontal distances of the vertex from the points of support x and x_1; let y and y_1 be given, and x, x_1 unknown.

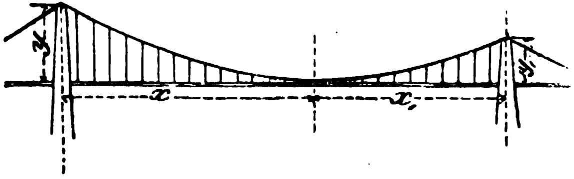


Fig. 35.

The horizontal stress at the vertex will be the same as if the bridge were composed of two symmetrical halves, each having a span $2x$ and a dip y, or of two symmetrical halves, with a span $2x_1$ and a dip y_1; in other words—

$$H = \frac{wx^2}{2y} = \frac{wx_1^2}{2y_1},$$

hence

6. . . . . $\frac{x^2}{y} = \frac{x_1^2}{y_1}$,

or,

$$x : x_1 = \sqrt{y} : \sqrt{y_1};$$

thus, to find the horizontal position of the vertex, we have only to subdivide the span in the ratio $\sqrt{y} : \sqrt{y_1}$; we may then calculate the strains on one side of the vertex as for half a bridge with the span $2x$ and the dip y, and on the other side of the vertex as for half a bridge with the span $2x_1$ and the dip y_1. The device of piers of different heights may be used with advantage when it is desired to throw a larger portion of the weight of the bridge on one pier than the other, because of a difference in the soundness of the foundations, or for other reasons. The stresses on the loaded and unloaded portion of the chains between the piers and the anchorage are easily determined by methods similar to those which have been given for the stresses on each part of the main span. The same methods also give the direction of each successive link, and of the final links leading to the anchorage.

§ 32. *Practical Details.* — The chains of suspension bridges are either long wire ropes or true chains made of links pinned together. Wire ropes allow the strongest known material to be adopted, namely, steel wire, which can be bought in large quantities of a quality which does not break with less than a stress of from 55 to 60 tons per square inch of section; charcoal iron wire of the sizes used will bear 40 tons per square inch; common sizes of wire for the purpose are from 0·16 to 0·14 inches, or say, No. 9 or 10 Birmingham wire gauge. Three or four thousand wires are not unfrequently used in one cable, and it is very essential that each wire shall take an equal part of the whole stress. It used to be thought necessary to ensure this by straining each wire separately either over the actual piers, or piers similarly placed, and binding them together when hanging, strained by their own weight with the dip proposed for the bridge. It was also thought essential that each rope should be an aggregate of parallel wires, not spun as in a hempen rope. Experiment has shown, however, that wire ropes spun with machines which do not put a twist into each wire, but lay it helically and untwisted, and with no straight central wire, are as strong as wire ropes of equal weight made with straight wires. They are, however, much more easily made. A number of ropes of this kind may, therefore, with more convenience and economy be bound together into one cable in the manner previously practised for single wires. Care should be taken to fill every interstice of the ropes with a bituminous compound.

When the chains are made of links of iron their ultimate strength cannot be taken as more than 30 tons per square inch, even if the very best material is secured. It is doubtful if this ultimate strength can at present be surpassed by steel links, for although many steel links of greater strength could certainly be obtained, occasionally a comparatively weak link will be produced even by the best manufacturers. In designing the links care must be taken to provide sufficient cross section at the eye, AB + CD, fig. 36, as well as at EF. The diameter of the pin BC must be such as will allow it to resist the shearing stress on it, and the surface of the pin and eye from B to C must be sufficient to bear the crushing stress. Otherwise, although the pin may not be shorn it may be squeezed flat, and the head of the link may bulge out and be much distorted under the stress. To obtain the necessary surface, without unduly increasing the diameter of the pin, the link may be rolled with a head broader than the body of the link. The section at GH and IJ must also be sufficient to resist shearing. When two or more parallel chains are used, care must be taken that the rods suspending the platform bear equally on the several chains. Fig. 37 shows a plan of securing this. Chains of unequal dip should not be used to support one platform, for the strain cannot be equally divided between them, inasmuch as they must deflect unequally with any passing load, or with any increase of temperature.

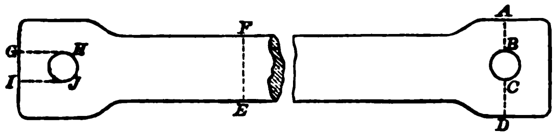


Fig. 36.

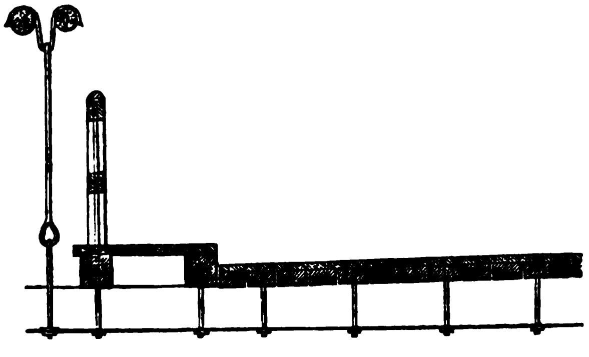


Fig. 37.

§ 33. *Merits and Defects of Suspension Bridges.*—The great merit of a suspension bridge is its cheapness, arising from the comparatively small quantity of material required to carry a given passing load across a given span. This merit may be easily seen by considering an elementary example. A man might cross a chasm of 100 feet hanging to a steel wire 0·21 inches in diameter, dipping 10 feet; the weight of the wire would be 12·75 ℔. A wrought iron beam of rectangular section, three times as deep as it is broad, would have to be about 27 inches deep and 9 inches broad to carry him *and its own weight.* It would weigh 87,500 ℔. An iron ⌶ beam of the best construction, 10 feet deep, would weigh about 120 ℔, without allowing anything for the stiffening of the centre web which would in practice be required. In each case four feet in length have been allowed for bearings at the ends of the span. The enormous difference would not exist if the beam and wire had only to carry the man, although even then there would be a great difference in favour of the wire; the main difference arises from the fact that the bridge has to carry *its own weight.* The chief merit of the suspension bridge does not, therefore, come into play until the weight of the rope or beam is considerable when compared with the platform and rolling load; for although the chain will for any given load be lighter than a beam, the saving in this respect will for small spans be more than compensated by the expense of the anchorages. In large spans the advantage of the suspension bridge is so great that we find bridges on this principle of 800 or 900 feet span constructed at much less cost per foot run than girder bridges of half the span. The disadvantages of the suspension bridge are, however, very great. A change in the distribution of the load causes a very sensible deformation of the structure; for the chain of the suspension bridge must adapt its form to the new position of the load, whereas in the beam the deformation is hardly sensible, equilibrium being attained by a new distribution of the stresses through the material. This flexibility of the suspension bridge renders it unsuitable for the passage of a railway train at any considerable speed. The platform rises up as a wave in front of any rapidly advancing load, and the masses in motion produce stresses much greater than those which could result from the same weights when at rest; moreover, the kinetic effect of the oscillations produced by bodies of men marching, or even by impulses due to wind, may give rise to strains which cannot be foreseen, and which have actually caused the failure of some suspension bridges. On the 16th of April 1850 a suspension bridge at Angers gave way when 487 soldiers were passing, and of these 226 were killed by the accident. Another danger peculiar to suspension bridges is that the platform may be lifted by the wind, when its oscillation will produce most dangerous strains. This accident may be prevented by tying the platform down to the piers or abutments. Lateral oscillation produced by the wind is also dangerous, and even gathered ice and snow may be a serious increment to the load on these bridges, forming a much more considerable fraction of the whole weight than where the supporting structure is itself massive. Suspension bridges must be well cross-braced to resist the action of the wind. They can be much stiffened laterally by placing the chains in inclined planes, converging downwards to the platform.

§ 34. *Modifications of the Simple Suspension Bridge.*—Many efforts have been made to design a bridge which shall combine the lightness of the true suspension bridge with the stiffness of the girder. Mr Dredge's design with sloping rods (fig. 38) gives a somewhat stiffer structure than the bridge with vertical suspension rods; the inclined rods throw a strain on the platform, which must be resisted either by ties along the central portion, or by struts abutting against the piers. The stresses on each part will be shown under the heading "Compound Structures" (§ 62).

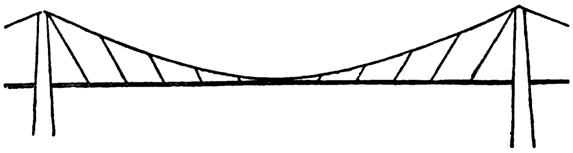


Fig. 38.

The design fig. 39 has been proposed by many, but is worthless. The object of the proposers is to support each part of the platform by rods which are quite independent of other parts of the structure, and which, being originally straight, do not alter their form under stress. The unequal stretching of the long and short rods under a stress, or with a rise of temperature, is a radical defect. Mr Ordish has proposed a plan in which the road is supported by sloping tie rods, arranged like the struts in fig. 87, inverted. Flexible chains, like that of an ordinary suspension bridge, carry the weight of these tie rods by vertical rods, which keep the sloping rods straight. The chain in this form is not subjected to unequal loading. Various forms of bridge have been proposed, in which, as in figs. 76 and 78, two chains are braced together. These may be made thoroughly stiff bridges, with a moderate increase in the amount of metal required for the flexible bridge. They will be described under the head of "Frames." Stiffness has also been obtained in some structures by using an auxiliary girder to stiffen the platform. This is best effected by the use for each chain of two girders, each half the length of the platform. These girders are placed as in fig. 40, being hinged together by a strong pin at B, and held down by pins at A and A_1, which should, however, be left free to move horizontally. These girders are not sensibly strained by the rise and fall of the chains due to a fall or rise of temperature; they can also deflect freely as a whole when the chain is deflected under strain; nevertheless, they serve to distribute the weight of a passing load over the chain, so that it cannot be sensibly distorted. Rankine has given the following rule for designing these stiffening girders. Let w_1 be the greatest rolling load per foot run; let x be the half span of the chain; let M be the greatest bending moment which the auxiliary girders will have to resist (*i.e.*, at the centre of each); let F be the greatest shearing force (at the end and central pins), then

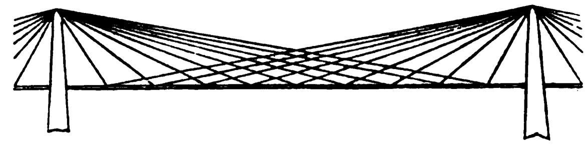


Fig. 39.

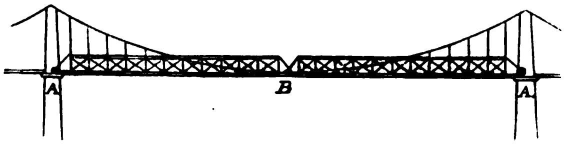


Fig. 40.

1. . . . . . $M = \frac{1}{16} w_1 x^2$,

and

2. . . . . . $F = \frac{1}{4} w_1 x$.

Each auxiliary half girder is in fact to be designed as a beam of half the span of the bridge, and capable of carrying half the passing load per foot run (but not its own weight). This plan of stiffening is quite effective, but adds considerably to the weight and cost of the whole structure; for not only have we to provide these extra girders, but extra material in the chains to carry this extra dead load.

§ 35. *Maximum Span.*—If we assume that wire can be obtained which will safely bear 15 tons per square inch, a rope (or single wire) with a dip of $\frac{1}{14}$th of the span would safely bear its own weight over a span of about one mile, and would not break till the span exceeded 4 miles. With a dip of $\frac{1}{7}$th of the span a steel wire rope of the best quality would not break until the span exceeded 7 miles. These lengths are not given as indicating practical spans for bridges, but to show the limits which with our present materials cannot be exceeded, however light the passing load may be.

IV. THE ARCH.

§ 36. *General Description.*—An arch may be of stone, brick, wood, or metal. The oldest arches are of stone or brick. They differ from metal or wooden arches, inasmuch as the compressed arc of materials called the *ring* (fig. 41, London Bridge), is built of a number of separate pieces having little or no cohesion. Each separate stone used in building the *ring* has received the name of *voussoir*, or archstone. The lower surface of the ring is called the *soffit* of the arch. The *joints*, or bed-joints, are the surfaces separating the voussoirs, and are normal to the soffit. A brick arch is usually built in numerous rings, so that it cannot be conceived as built of voussoirs with plane joints passing straight through the ring. The bed-joints of a brick arch may be considered as stepped and interlocked. This interlocking will affect the stability of the arch only in those cases where one voussoir tends to slip along its neighbour. The ring springs from a course of stones in the abutments, called *quoins*. The plane of demarcation between the ring and the abutment is called the *springing* of the arch. The *crown* of the arch is the summit of the ring. The voussoirs at the crown are called *keystones*. The *haunches* of the arch are the parts midway between the springing and the crown. The upper surface of the ring is sometimes improperly called the extrados, and the lower surface is more properly called the intrados. These terms, when properly employed, have reference to a mathematical theory of the arch little used by engineers. The walls which rest upon the ring along the arch, and rise either to the parapet or roadway, are called *spandrils*. There are necessarily two outer spandrils forming the faces of the bridge; there may be one or more inner spandrils. The *backing* of an arch is the

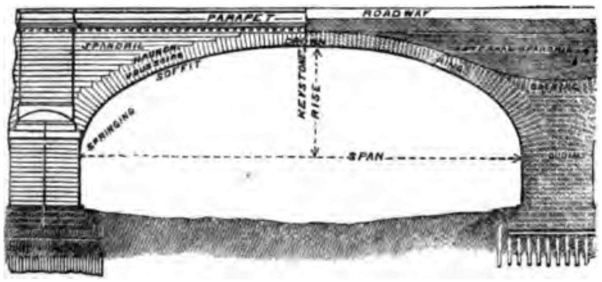


FIG. 41.—Half Elevation and Half Section of Arch of London Bridge.

masonry above the haunches of the ring; it is carried back between the spandrils to the pier or abutment. If the backing is not carried up to the roadway, as is seldom the case, the rough material employed between the backing and the roadway is called the *filling*. The *parapet* rests on the outer spandrils. The abutments and piers have the same signification as in other bridges. The masonry arch differs from the superstructure of other bridges in the following respect: it depends for its stability on the presence of a permanent load specially arranged, and so considerable in amount that the changes produced in the direction and magnitude of the stresses by the passing load are insignificant. The theories of the masonry arch often neglect the passing load entirely, and simply teach the student how to distribute the permanent load, so that the voussoirs may be in equilibrium. The permanent load consists of the ring, the backing, the filling, the spandrils, and the roadway. Inasmuch as the ring is that part of the structure which by its special strength and arrangement carries the superstructure in the same sense as a beam or chain carries it, the arch in this article will be treated simply as a ring of voussoirs springing from two abutments and loaded with weights, some permanent and some passing. Where the backing strengthens the arch, it becomes virtually part of the ring.

§ 37. *Equilibrium of a Single Voussoir.*—A block, such as a voussoir, ABCD, fig. 42, resting on one of its surfaces, such as the joint AB separating it from the next voussoir, is in equilibrium when the resultant of all the forces acting upon it (including its own weight) falls within the supporting surface, while the direction of this resultant makes an angle ϕ with the normal to the surface less than the angle of repose; (the tangent of the angle of repose is the coefficient of friction). If the resultant, as $R_{,}$, falls without the surface, the block will heel over, pivoting on the edge A. If the resultant, as $R_{,,}$, although falling within the surface of which AB is the trace, is yet much inclined to the normal, the block ABCD will slide up on the joint AB without heeling over. The block, if used as the voussoir of a bridge, must not only be in equilibrium under the forces applied to it, but must also be of sufficient strength to resist these forces. The intensity of crushing stress due to the external forces must nowhere exceed the safe crushing strength of the material. This latter condition would in most arches be fulfilled by an extremely thin ring of stones or brick if the resultant passed through the geometrical centre of the joint AB in a direction normal to it. In that case the stress on the joint would be a uniformly distributed stress; if, however, the resultant stress passes near one edge, the intensity of stress at that edge will be much greater than elsewhere, and would indeed be infinite if the resultant passed exactly through the edge at A or B; while, therefore, the condition of equilibrium is satisfied if the resultant passes within either edge of the voussoir at no great inclination, the condition of strength requires that this resultant shall not cut the joint very near the edge, and the common practical rule is that it shall always fall within the middle third of the joint. This rule is based on the condition that the pressure on a joint shall nowhere be negative; in other words, that no tension shall occur at any part of any joint. The principles explained in § 8 show that the minimum stress on any joint $p_1 = p_0 - \frac{Px_0x_1}{I}$, or that the stress will be zero, when $p_0 = \frac{Px_0x_1}{I}$. Let d be the depth of the rectangular joint, and b the breadth; then

$$I = \frac{bd^3}{12}, \text{ and } x_1 = \frac{d}{2}, \text{ and } p_0 = \frac{P}{bd};$$

hence

$$\frac{P}{bd} = \frac{12Px_0d}{2bd^3}, \text{ or } \frac{d}{6} = x_0,$$

an equation expressing the condition that the centre of pressure lies at the edge of the middle third; any greater value of x_0 will give a negative value to p_1. We shall see that the actual resultant is, according to the theory practically in use, indeterminate within certain limits; it is therefore useless to attempt to calculate the exact maximum stress on any one stone. In the rest of this article the *ring* is to be held to mean the *middle third* of the actual masonry, or brick ring, wherever the theory requires that the blocks are to resist practical loads. As bridges are subject to a sensibly equal load on all parts of their breadth between the parapets, it is usual to consider a portion of the ring one foot in width, each other strip being under precisely similar conditions. Similarly the joint may be spoken of for convenience as the line which is its trace, and the edge as the point which is its trace.

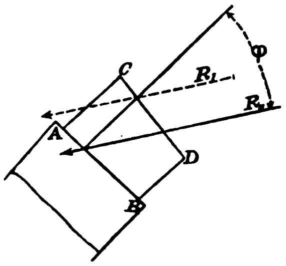


Fig. 42.

The external forces which act on any voussoir are—1st, the vertical force, being the resultant of its own weight and the load which is directly over it; 2d, the thrust from the voussoir above it; and 3d, the reaction from the voussoir on which it rests (fig. 43). It is sometimes difficult to determine exactly what portion of the superincumbent load a voussoir may properly be said to carry, but a sufficient approximation is obtained for practical purposes by assuming that the mass vertically above any voussoir is carried by the voussoir when the back of the voussoir is not much inclined. If the materials had little cohesion, the direction of the force produced by the load would not be vertical, but inclined at an angle depending on the coefficient of friction; in practice, the direction of the force is uncertain and even variable with changes in the condition of the superincumbent filling. If, however, the stability of the arch is calculated with a reasonable margin or coefficient of safety, on the hypothesis that the force produced by the load is vertical, there is every probability that the arch will be stable under any actual stress which may arise in practice.

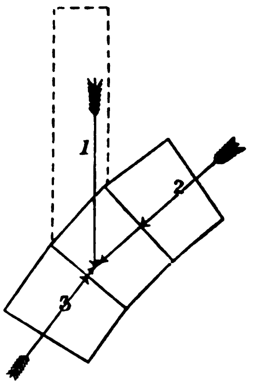


Fig 43.

§ 38. *Equilibrium of any three Voussoirs; Equilibrated Polygon.*—The simplest arch would be an arch of three voussoirs resting on two abutments, and any actual arch consisting of many voussoirs may be considered as composed of successive triplets, the voussoirs on each side of which act as abutments. If, therefore, we can show the conditions of equilibrium for three voussoirs we shall have determined the conditions for the whole ring.

Let three voussoirs be taken from any part of the ring (fig. 44), and let the lines 1, 2, and 3 represent the position of the resultants of the three known loads w_1, w_2, and w_3 (including the weight of the voussoirs) borne by each voussoir.

Let NA represent the position of the reaction t due to the abutting voussoir on one side. Let A be the point where the prolongation of the line NA cuts the line 1; then if the magnitude of the forces t and w_1 are known, these determine the magnitude and direction of the equilibrating force t_1, which must act at A to balance them. Let the direction AB and the magnitude of the force t_1 be

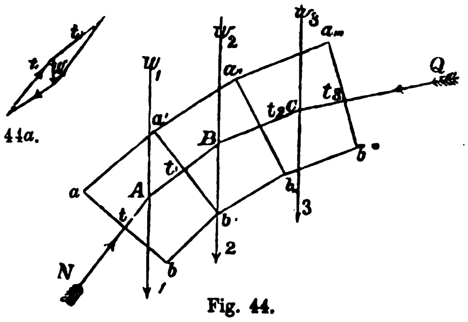


Fig. 44a.

Fig. 44.

found by the ordinary parallelogram of forces, as in fig. 44a, and let B be the point of intersection of the direction of this force with the line 2; then the direction and the magnitude of the equilibrating force t_2 can be found as for t_1; similarly the direction of this force gives the point C by its intersection with the line 3, and finally we obtain, by the resolution of forces, the direction, magnitude, and position of the force t_3, by means of which the reaction of the second abutment will keep the system in equilibrium. When the position and magnitude of t are known, the position and magnitude of all the other forces are determinate; the conditions of equilibrium are that the lines NA, AB, BC, and CQ, shall not cut the joints above or below the edges a or b, for in that case the blocks would heel over on the edge beyond which the resultant passed; also, the direction of the lines NA, AB, &c., must be such as not to exceed the angle of repose with the normal to the joints, otherwise one stone will slip on the other. The abutment producing by its reaction the force t_3 must not yield with a less force than t_3, and must not be pushed forward so as to produce a greater force than t_3. The line NABCQ, if inverted, is in form identical with that which a cord would assume, loaded at the points A, B, and C, with the loads 1, 2, and 3, and having the direction of NA determined. This line will, in the rest of this article, be called an *equilibrated polygon*

When the joints are supposed indefinitely near, or the voussoirs thin sheets, the equilibrated polygon becomes a curve called a *linear arch*.

The reasoning applied to three blocks is clearly applicable to any number, and we may therefore say that any series of loaded voussoirs will be in equilibrium when a reaction of known magnitude and direction is applied at one abutment, provided the equilibrated polygon required by this reaction and the given loads can be drawn so that its sides cut all the joints within the ring (or within the middle third, where strength is an element of the question, at an angle greater than the complement of the angle of repose for the material used. An equilibrated polygon, ABC… ..Q, for a complete arch is shown in fig. 45 Fig. 45a is the diagram giving the slopes KJ, JI, &c., as for the loaded chain fig. 30 and 31

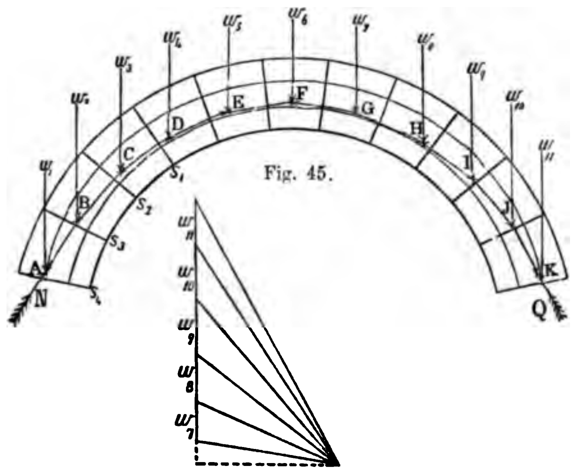


Fig. 45.

Fig. 45a.

§ 39. It will be shown in next paragraph that the arch will be in equilibrium if with any value of the horizontal thrust h an equilibrated polygon can be drawn fulfilling the conditions required. In most arches equilibrated polygons fulfilling these conditions can be drawn with values of h varying between two limits differing by a considerable amount. In that case the smallest value of h will be the true value, and give the true stresses; for the abutments being inert will not give back a greater thrust than is just required to balance the structure. This would render the thrust h determinate if the equilibrated polygon might actually approach the true edge of the ring, but as this would require infinitely strong materials, we are still left in uncertainty as to the true value of h, but may feel sure that it will be the smallest value consistent with a safe stress on the material. If we provide abutments capable of reacting with a force h sufficient to keep the equilibrated polygon (where it cuts the joints) within the middle third, our abutment will certainly be amply strong enough, and this is the value of h to be adopted in all practical calculations of the stability of an arch.

In the example with the three voussoirs it is clear that equilibrium would be obtained with very widely different reactions t at the one abutment. And this fact is also true for a bridge of many voussoirs. The vertical component (which may be called v) of this total thrust t is indeed determinate if we suppose the point where t cuts the joint to be known, being the same as the vertical reaction from a beam carrying the same weights and supported at the points where t and v cut the abutments; but the horizontal component or horizontal thrust, which has been called h, cannot be determined by any considerations hitherto mentioned.

§ 40. *Experimental Demonstration that the Equilibrium of a series of Voussoirs is stable if any Equilibrated Polygon can be drawn fulfilling the conditions stated above.*—Let us suppose an arch, fig. 48, to be constructed, the bed-joints of which are not plane but curved, so that each stone touches its neighbour only along a horizontal line, the trace of which in a drawing may be called the *point of contact.* Such an arch will differ from an ordinary arch in this respect, that the centre of pressure at joints will be shown by the points of contact, while the stones will be able by rolling to alter the points of contact if not in equilibrium. In such an arch the voussoirs in the first place may be put together so as to touch at any desired series of points, but the forces called into play when external support is withdrawn will rearrange the voussoirs so as to bring them into equilibrium, if any equilibrated arch consistent with the loads can be drawn so that the lines forming it cut the joints inside the ring, and a model will show the points of contact, or, in other words, the places where these lines cut the joints. (It is assumed that the obliquity of the sides of the polygon to the joints which they cut is insufficient to produce slipping.)

An actual model shows the action very prettily, but the following considerations will easily allow the student to see how it is that the voussoirs always arrange themselves so as to build a true arch.

Suppose, first, that the arch consisted merely of three stones, fig. 46, and that the weight on the centre one was so great that the linear arch, or equilibrated polygon, became sensibly two inclined straight lines like rafters. As soon as the voussoirs are left to themselves, the pressure at the surface a_2 b_2, and the reaction at the surface a_1 b_1, will lie in one straight line, which, meeting a similar straight line from the other abutment, will give one equilibrated polygon, satisfying the required conditions; but if the horizontal force required for this polygon is not supplied by the abutments, the two forces at joints 1 and 2 will, as shown by the small straight arrows, constitute a couple tending to turn the stone A round, so that the point of contact at joint 1 will be lower, and the point of contact in joint 2 will be higher than before. The same action will occur in stone C, and the result will be that the weight may be balanced with a smaller horizontal force. At the same time the rotation of the stones A and C, coupled with the descent of B, tends to push back the abutments N and Q, and therefore to increase their horizontal reaction supposing them to be stable. If the abutments N, Q continue to yield, the stones A

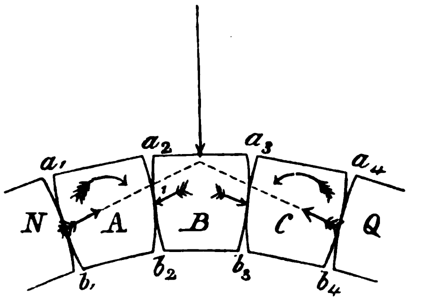


Fig. 46.

and B will continue to turn until the points of contact reach a_2 and a_3, or b_1 and b_4. The horizontal thrust which the abutments require to meet will therefore diminish as the stones turn, and the little structure will only fail to support the weight in case the abutments N and Q are insufficiently strong or stable to supply the minimum thrust consistent with an equilibrated polygon

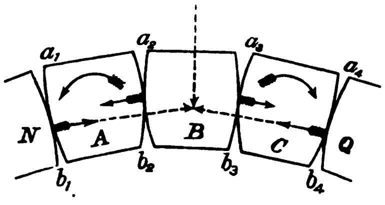


Fig. 47.

cutting the joints inside the ring (or in case the polygon cut the joints at such an angle that the stones slip). If, on the other hand, the abutments were so made as to press in upon A and C with a greater horizontal force than is consistent with two lines of pressure passing through the actual points of contact, then, as in fig. 47, the direction of the couples on the stones A and B would be reversed, and they will roll round so as to bring the points of contact more nearly into the position required to meet an excessive horizontal thrust, and at the same time the changed position of the stones, by allowing N and Q to come forward, will tend to relieve or diminish the original excessive horizontal thrust, where this is due to the elasticity of the stones N and Q, or of the stones supporting these abutments. The structure will not fail unless the points of contact reach a_1 and a_4, or b_2 and b_3, when the structure would fail by the sides being squeezed in, and the stone B being lifted up out of the arch. This could not happen with stones of the proportions shown in fig. 47, as before the limiting position was reached, the points of contact would lie on a straight line corresponding to an infinite horizontal thrust. In conclusion we see that, whether the horizontal force supplied by the reaction of the abutting stones be too small or too great, the three voussoirs tend to move so as to adapt the centre of pressure and the actual horizontal force to one another. The equilibrium produced is stable, that is to say, if by some external force the arrangement of the blocks is slightly disturbed, when the force is removed the blocks return to their original position. In the above demonstration it is assumed that the blocks when first put together touch at some point not far from the centre of the bed,—a condition corresponding to reasonably good fitting in the case of the plane joints of a stone arch before the centring is removed.

If a model be prepared (fig. 48), having a number of voussoirs of wood with their bed-joints slightly curved and roughened, the result of the above theory will be very clearly and beautifully seen. The action explained in the case of three blocks holds good for any three, and therefore for the whole series. If an additional weight is placed at the crown, as in fig. 48, the crown is a little lowered, but the curve passing through the lines of contact rises at the crown and is lowered at the haunch by the rotation of the blocks, until the lines of contact at the joints arrange themselves, so that the resultant pressures forming the imaginary polygon pass through these lines of contact. If the extra load be placed at the haunches the crown rises, but the points or lines of contact between the voussoirs are lowered at the crown and raised at the haunches, as in fig. 48a. If one haunch only is weighted, the curve passing through the lines of contact rises at that haunch and is lowered at the other, as in fig. 48b; if the model be distorted by the hand it oscillates up and down on each side of the position of equilibrium, as a string similarly loaded would do. Figures 48, 48a, and 48b are taken from photographs of a model. (It should be remarked that the abutments were screwed to the supporting board; it is obvious that otherwise they would not have been in equilibrium.) The general character of the curve passing through the points of contact may be easily conceived by thinking of a string similarly loaded and inverted. The equilibrated arch will be one of those forms which a string might take when similarly loaded, but when the load is changed, the length of the curve will not be constant in the arch, whereas it must be constant with any given chain. The curve passing through the points of *contact* corresponds with what Moseley called the line of resistance. The direction of the pressure is not necessarily tangent to this curve, but in the ordinary form of bridge it is nearly so.

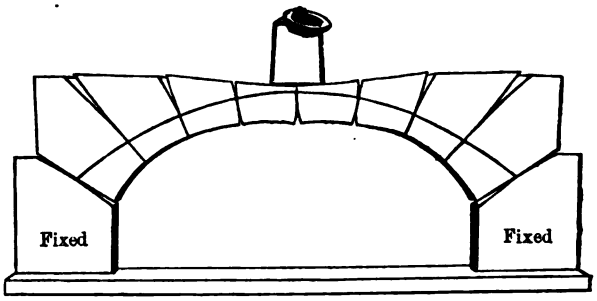


Fig. 48.

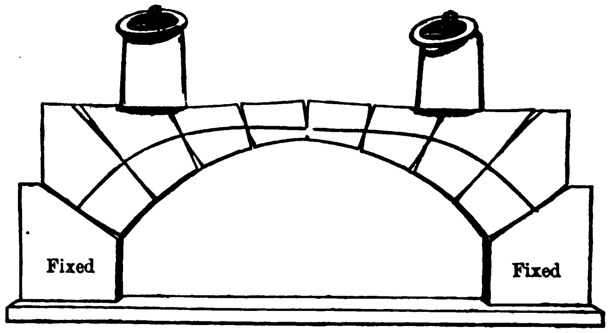


Fig. 48a.

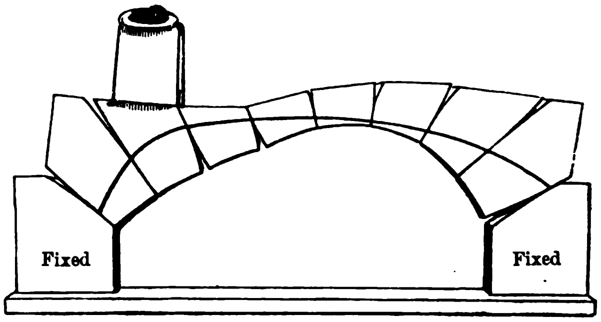


Fig. 48b.

In the model each voussoir is free to roll, because the bed-joints are curved. In an actual bridge the bed-joints are plane, nevertheless, the stones do turn round to adapt themselves to the pressure, but the result of this rotation is to render the compression along the upper and lower halves of the stone unequal. One edge is more compressed than the other; the couple tending to turn the voussoir, and actually allowed to do so in the model, is met by an equal and opposite couple, due to the unequal compression of the stone.

This couple is the necessary result of a pressure which is not axial, *vide* § 8; an equilibrated polygon cutting the joints at various distances from the centre is therefore as correct an indication of the actual forces present in a practical arch with flat joints as in the model with curved joints; but we must remember that where the joints are flat, the pressure will be unequally distributed wherever the line of the equilibrated polygon does not cut the centre of the joint. Greater or less elastic resistance in the stone corresponds to greater or less curvature in the surface of the joint. A small distortion of the arch will restore equilibrium when the curvature is great, or when the stone has a high modulus of elasticity. The ring with plane bed-joints is in stable equilibrium, and adapts itself to new distributions of load for precisely the same reasons as the model with curved joints, but in the one case the couple called into play to move the voussoir is actually cancelled by the new position which the points of contact assume; in the other case it is balanced by the equal and opposite couple resulting from the resistance to motion due to the hardness of the stone.

The preceding paragraph showed how to determine whether an arch was in equilibrium when a known reaction was applied at one abutment; the experiment and reasoning now given show that the incipient yielding of an arch under loads will produce a reaction at the abutments suited to keep the whole ring in equilibrium, provided only an equilibrated polygon can be drawn, cutting the joints within the ring at suitable angles.

§ 41. *Practical Investigation of the Stability of a given Arch under a given Load—Joint of Rupture.*—This investigation resolves itself into finding that equilibrated polygon or linear arch which can be drawn within the (middle third of the) ring from the crown to the *lowest possible joint* of the ring (or to the springing if this be possible). This lowest possible joint must in any case be treated as the springing of the arch, and if the linear arch goes out of the (middle third of the) ring above the actual springing, as will be the case in all semicircular or elliptical rings, masonry must be provided in the backing capable of taking the actual thrust into the abutment and constituting the real arch, which often differs widely from the form indicated by the ring of stones in the face. The linear arch in a circular or segmental bridge loaded simply by its own weight generally has a smaller radius of curvature than the ring at the crown, and a much larger radius towards the haunches. Consequently, the longest linear arch which can be drawn within the ring will approach the upper surface of the ring at the crown and the soffit towards the haunches.

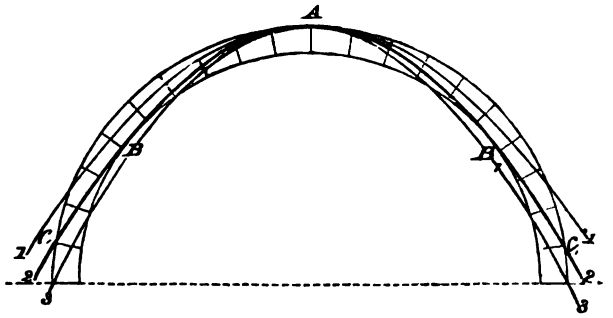


Fig. 49.

Fig. 49 shows a series of linear arches, all drawn for the same load, and all tangent to the upper surface of the crown of the arch, differing only in being the result of different horizontal thrusts. The curve drawn with a thick black line, tangent to the soffit, is clearly the longest linear arch which can be drawn within the ring. Any smaller value of the horizontal thrust h would give a linear arch like curve 3, and any larger value of h would give a linear arch like curve 1, and both these values of h are incompatible with equilibrium for the whole arch down to joint C; if, therefore, the arch fails by the yielding of the abutment, or of the lower portion of the ring, the failure will first be apparent at the joints A and B, where this black line is tangent to the ring, and at joint C, where the linear arch cuts the back of the ring. Smaller values of h will keep the stones in equilibrium above and below joint B, but unless the arch below the joint B, as well as the abutment, can resist the tendency of the arch to spread, or, in other words, supply *at least* the horizontal reaction h required for this linear arch, the joint B will open at the top, the centre joint A will open at the bottom, the joint C will open at the back, and the crown fall in as shown in fig. 49*a*. The joint B, where the longest linear arch is tangent to the soffit, is called the *joint of rupture*. The value of h required to make a linear arch tangent to the back of the ring at the crown pass through the edge of the joint of rupture at the soffit, is larger than the value of h required to give a linear arch passing through the edge of any other joint at the soffit; at the same time, it is the smallest value of h consistently with which the arch can remain in equilibrium down to B and from B to C. In circular arches the joint of rupture generally makes an angle of about 30° with the horizontal plane; in elliptical arches the angle is usually about 45°. Its position is easily found as follows:—Let y_1, y_2, y_3, &c. (fig. 50), be the heights of the upper surface of the crown A above any points B_1, B_2, B_3 at the lower edges of the soffit; let W_1, W_2, W_3 be the weights of the portions of the arch with its load carried by the ring from B_1 to A, from B_2 to A, from B_3 to A, &c. (The load is in the fig. assumed to be symmetrically disposed relatively to the centre of the span.) Let x_1, x_2, x_3 be the horizontal distances of the centres of gravity of w_1, w_2, w_3 from the points B_1, B_2, B_3, &c.; then taking moments round B_1, B_2, B_3 in succession, we have, if the linear arch be assumed to pass through any point B—

$$Wx = hy;$$

taking the successive values of h for a series of joints B, we shall find that one joint gives a maximum value. This value corresponds with that of the linear arch tangent to the soffit (of the middle third) at the joint of rupture; for this arch has the maximum thrust of any passing through the points B_1, B_2, &c., as appears by simple inspection of fig. 49. The joint of rupture can thus be tentatively found, and the value of h, or the thrust which the abutment must resist, is obtained at the same time. If the backing is carried well up above C, a larger value of h than that obtained by this method would be consistent with the stability of the arch, and might actually occur; but we need not provide for this larger value, since the yielding of the abutment under it would diminish the thrust till it fell to the value as above determined. If the abutments could resist this thrust, the bridge would then remain in equilibrium. If the arch is flat there may be no joint of rupture, and in that case the value of h is to be taken as that given by a linear arch passing through the bottom of the (middle third of the) springing and tangent to the crown of the arch, *i.e.*, to the summit of the middle third of the ring.

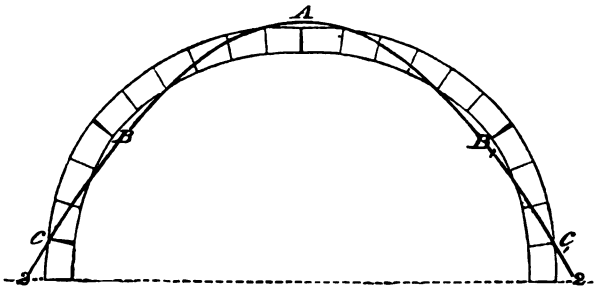


Fig. 49*a*.

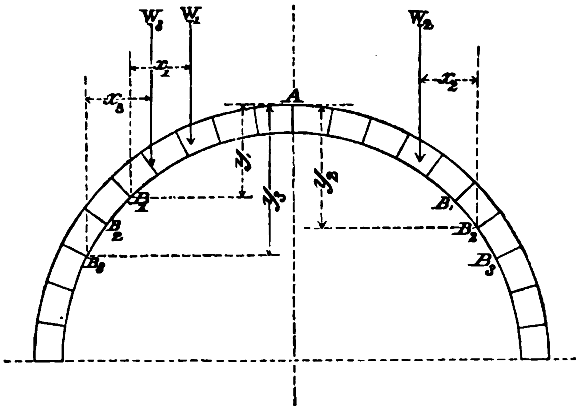


Fig. 50.

When the apparent springing lies much below the joint of rupture, we find that the linear arch leaves the ring on the upper surface at a joint (C) lower down, where failure must result by the opening of the joint at the lower surface, unless the pressure is taken by masonry outside the ring. It is for this purpose that the *backing* is required. Obviously the best mode of supplying backing is to thicken the ring itself, keeping the masonry joints radial. The portion of the arch below the joint of rupture B is often considered as part of the abutment.

If the load at the crown of an arch were very light, and the load at the haunches comparatively very heavy, the series of tentative curves drawn with various values of *h* would assume the character shown in fig. 51. The longest curve which can now be fitted into the ring (drawn with a thick black line in the figure) will probably approach the soffit at the crown and be tangent to the upper surface of the ring at the haunch. In fig. 51 the longest linear arch is shown as tangent to the soffit at C. This condition could seldom be secured; with most loads the linear arch tangent to the back of the ring at B will *cut* the soffit at C. Nevertheless, the value of *h* to be provided for will be that given by the linear arch tangent to the soffit. If this arch leaves the middle third at B, the ring must be thickened or efficient backing provided at this point. If the abutment yield an arch thus loaded would fail, as in fig. 51*a*, but the case very seldom arises in practice. If the arch were not pointed at A, but curved so as to contain the linear arch near the crown, the piece BAB would be lifted up as a whole without breaking at A.

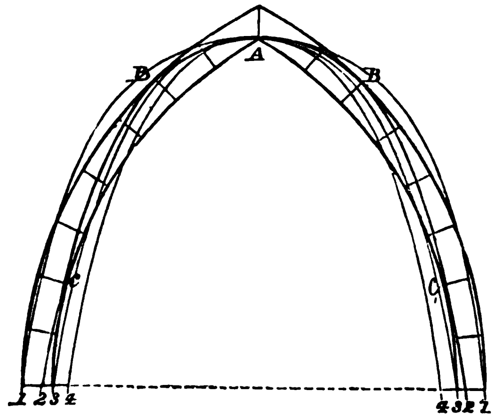


Fig. 51.

The joints of rupture can be found for unsymmetrical loads as well as for symmetrical loads, but these joints will then not be at equal distances from the crown.

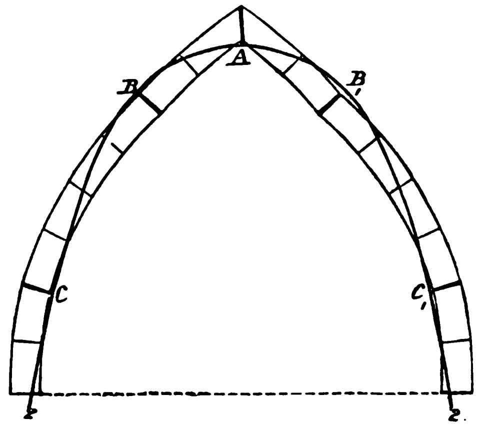


Fig. 51*a*.

If the middle third of the ring be alone treated as effective, the designer, after finding the joint of rupture for a bridge of the usual form and with usual loads, need make no further calculation as to the arch *above* that joint. A linear arch which is tangent to the soffit at the joint of rupture, and to the upper surface of the ring at the crown, will probably lie within the ring at intermediate joints, and will cut them at an angle not differing much from a right angle; but the linear arch must be carried on below the joint of rupture, through the backing and the abutments, to see that it is nowhere too much inclined to the bedding joints, and never comes too near the edge of the effective masonry. The horizontal thrust determined by finding the joint of rupture on the hypothesis that the middle third of the ring is the only effective part will be a safe value; but the actual value may be considerably less, since the actual linear arch called into play may lie outside the middle third. Since we do not know the actual position of the resultant pressures on each voussoir, any refinement in calculating the maximum intensity of stress due to these resultants would be useless. If the actual horizontal thrust were known, it would be easy to determine the couple acting on each joint and due to the distance between the resultant pressure and the centre of resistance of the joints; then knowing this couple and the total thrust it would be equally easy by the principles in § 8 to determine the maximum intensity of stress. Practically the thickness of the arch ring is determined by rules derived from experience, and the chief use of the above theory is to determine the dimensions of the abutments; if, however, with a given load the joint of rupture were found much nearer the crown than the positions indicated above, it would be well to rearrange the permanent loads or to alter the form of the ring.

§ 42. Professor George Fuller of Belfast has communicated the following novel and very neat method of finding the linear arch of maximum rise (and therefore of minimum thrust) which can be drawn within the middle third of a given ring.

In fig. 52 let the dotted curves GI and HK bound the middle third of the ring. Let the span be divided into any convenient number of parts at *a*, *b*, *c* . . . &c. Let the load on the half arch be subdivided into a corresponding number of parts, and each partial load referred to the vertical line passing through *a*, *b*, *c* . . . &c. Let the curve D 1 2 3 . . . A be a curve of bending moments for these loads, drawn to any convenient scale. This curve will also (§ 30) be a linear arch for the given loads. Draw the straight line AB at any convenient inclination, cutting the horizontal line DB at B. Raise the verticals *a*1, *b*2, *c*3, . . . &c., from the points 1, 2, 3, &c. Where these cut the curve DA draw horizontal lines, cutting AB at 1′, 2′, 3′, . . . &c. Since the ordinates of all possible linear arches are merely multiples or submultiples of the curve of bending moments, it follows that any other straight line from B to the vertical through A will have ordinates, which, if measured from DB along the verticals passing through 1′, 2′, 3′, &c., will be the ordinates of a linear arch, set off on the corresponding verticals passing through 1 and *a*, 2 and *b*, 3 and *c*, &c. AB might be called the development of the linear arch DA. Now let the curves CI and HK be developed in a similar way, so that, for instance, the ordinates measured from *a* to these curves are equal to the ordinates measured on the vertical passing through 1′ from DB to the developments I_1 G_1 and H_1 K_1; then it is clear that for the given loads any linear arch which lies within the middle third of the ring must, when developed, be represented by a straight line lying within the area I_1 G_1 H_1 K_1, and consequently that the straight line BC, which starts from the lowest point B in this area, and is tangent to the curve G_1 I_1, will be the development of the curve of

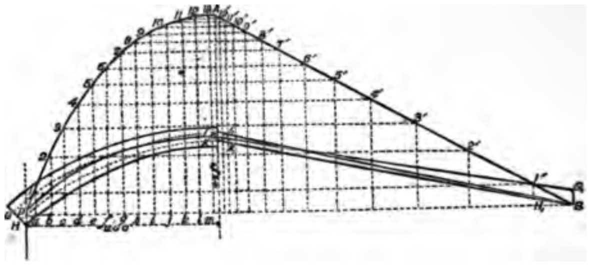


Fig. 52.

maximum rise and minimum thrust which can be drawn within the given middle third with the given loads. The line BC determines the point C, and the ordinates of BC give the ordinates of the curve DC, the ordinates being measured on corresponding verticals.

The example shown is the diagram for an arch of 52 feet span and 10 feet 4 inches rise, the depth of the ring at the crown being 2 feet 6 inches, and at the springing 3 feet 8 inches. The loads per foot of breadth, beginning at *a*, are—23·04, 19·45, 16·35, 13·94, 12·01, 10·35, 8·98, 7·93, 6·32, 5·79, 5·52, 5·39 cwts The rise of the linear arch found is 10·2 feet, and *h* per foot of breadth of the arch 129 cwt. $= \frac{1315 \text{ ft. cwt.}}{10{\cdot}2 \text{ ft.}}$

§ 43. *Empirical Expression for the Thickness of the Ring.*—The ring when not of equal thickness is always made of least depth at the crown. The depth of the key stone is therefore the thickness of the ring at its smallest part.

Let D be this depth in feet, and *r* the radius of the arch in feet at the crown. Then we may take (Trautwine)—

$$1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad D = C\sqrt{r}.$$

According to Rankine, C may be taken as ·346 for a single arch, and ·413 for one of a series of arches. The reason for making one of a series thicker than a single arch is, that the former has, when not loaded itself, to bear part of the thrust from its neighbours when these are loaded; this thrust tends to throw the linear arch in the unloaded span low down in the keystone. The following is another series of values of C in practical use :—

For first class stonework ....C = ·36
,, second class stonework ....C = ·4
,, brick and rubble.......................C = ·45

Perronet gives the following rule :—Let L be the span in feet—

$$2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad D = 1 + \frac{L}{30}.$$

Rankine, *Civil Engineering*, p. 427, shows that Trautwine's rule is rational. Perronet's can only be so when the usual proportion of rise to span is adopted.

Brickwork arches of 24 feet span and less are made 1 foot 6 inches deep at the crown; 30 feet span, 1 foot 10½ inches; 40 feet span, 2 feet 3 inches. The usual flat arch of these dimensions has its ring increased by two rings of bricks towards the haunches. These do not show on the face being concealed by the spandrils. Rubble arches are made a little thicker.

§ 44. *Practical Details.*—The strongest and simplest form of arch is a flat circular arc, having a rise of about one quarter of the span. In these arches the springing is above the place where the joint of rupture would occur if the ring were prolonged. Those parts of an elliptical or semicircular arch which lie below the joint C, fig. 49, are of use chiefly to improve the appearance of the arch. They are virtually part of the abutment, which is sometimes even considered as extending to the joint B. In a very flat arch the linear arch may be brought to coincide more truly with the axis of the ring by lightening the haunch, with which object the roadway is sometimes carried on small flat arches turned at right angles to the main arch, and having the spandrils of the main arch as abutments.

The joints between the voussoirs should be very evenly worked, so that the pressure may be evenly distributed. In brick joints the layers of mortar should be thin. Great care should be taken to provide for the drainage of the roadway above the arch. With this object the masonry should be covered with a sheet of asphalt sloping down to the piers or abutments, and suitable drains must be provided to collect the water and discharge it through the pier or abutment.

Skew Arches have already been treated of under the general head **Arch** (vol. ii. p. 330).

Considerable attention must be given to the construction of the *centres* or wooden frames on which the voussoirs rest while the ring is in process of being built. Extreme rigidity is necessary, and this rigidity is best attained by adopting one of the three following plans (Rankine):—1. Direct supports as in fig. 53, illustrating Hartley's centre for the bridge over the Dee at Chester (total span 200 feet); 2. Inclined struts in pairs as shown in fig. 54, being a diagram of the centre used in the erection of Waterloo Bridge; 3. Trussed wooden girders, of which an example is afforded by the truss used in the erection of London Bridge, fig. 55.

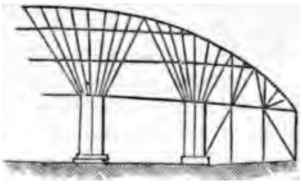


Fig. 53.

Figure 55 shows the *striking plates* and *wedges* by which the centre is lowered after the completion of the arch. The upper and lower plates A and B are strong beams suitably notched, and are separated by the compound wedge C; this wedge is kept in its place by cross wedges shown in section in the figure. When the centre is to be lowered these cross wedges are knocked out, and the main wedge C driven back. Owing to defective centering some large French arches sank much during construction, and owing partly to this cause, and partly, as it would appear, to defective mason-work, the total deformation after the centres had been struck was most extraordinary. In Perronet's bridge at Neuilly (*vide* Table XVII., § 84) the sinking, while the centre was in its place, amounted to 13 inches, and after the centre was struck a further sinking took place of 9½ inches. The crown of the centering had a radius of 150 feet, but the sinking of the arch was such that for 60 feet it assumed the form of an arc of a circle with a radius of 244 feet. It is remarkable that the bridge, built in 1774, of very bold design and so imperfectly executed, still stands. When the centres of Waterloo Bridge were removed no arch sank more than 1½ inches. Centres have occasionally been supported on strong sacks full of sand. To lower the centre the sand was allowed to escape through apertures in the sack. It is believed that this method was first employed by a French engineer, M. Beaudemoulin. The canvas sack has been advantageously replaced by wrought iron

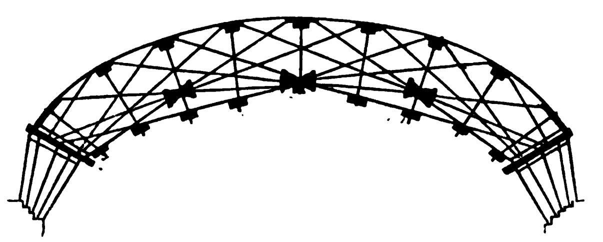


Fig. 54.

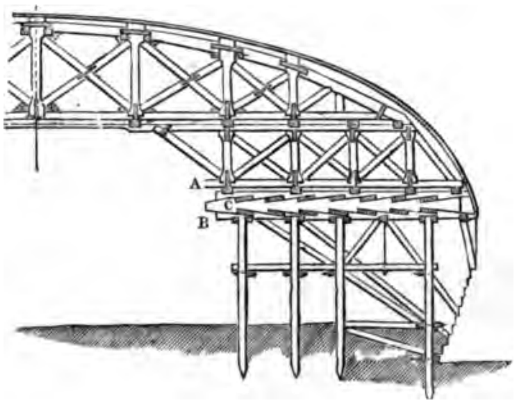


Fig. 55.

boxes or troughs; the block supporting the centre acts as a lid resting on the sand inside; when the sand is allowed to escape the block sinks slowly down inside the box.

§ 45. *Comparison of Metal with Masonry Arches.*—Metal arched ribs may be used instead of rings of masonry to support a platform and roadway. These arched ribs constitute true arches whenever, as is generally the case, all parts of the rib are compressed. The principles by which the stress on each part may be computed do not differ from those already explained for arches of masonry, but it is possible to calculate the stresses with much greater exactitude for continuous metal ribs than for voussoirs. With voussoirs we have seen that the resultant thrust at the springing is indeterminate both in magnitude and position, but we shall see hereafter that the resultant thrust, which will be called *t*, at the springing of a metal arch is easily rendered determinate. Supposing *t* and t_1, the thrusts due to a given load (fig. 56), to be known, then if the form of rib be made to correspond with any linear arch for the given distribution the compression at any section of the rib will be axial and uniformly distributed; the arch will then be strained as a chain of the same length would be strained under the same distribution of loads, extension being substituted for compression and dip for rise. Fig. 56 shows a rib of this kind with the approximate linear arch drawn as an equilibrated polygon by the method explained in § 38.

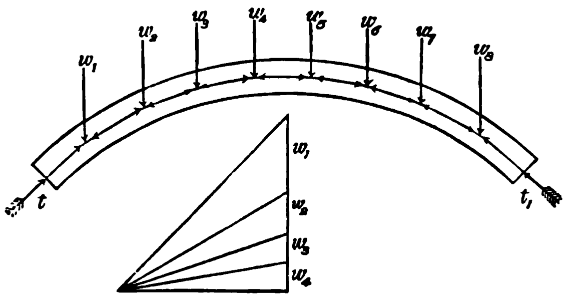


Fig. 56.

If the distribution of the load is altered the linear arch will also change, and the stress on each part of the rib will no longer be axial. The change in the form of the linear arch will generally be much greater for a metal than for a masonry arch, because most metal arches have light open spandrils and a light roadway, so that the passing load is considerable in comparison with the permanent load. Not improbably the linear arch, when only one haunch of a metal rib is loaded, may pass quite outside the rib for a portion of its length if this rib is made, as is usually the case, of a form containing the linear arch for a symmetrically distributed load. On the other hand, it does not follow, as with masonry, that because the linear arch passes outside the rib the bridge will fail. The bending couple then produced can be resisted by the moment of the elastic forces of the cross section of the rib if the rib is made strong enough. In masonry the joints open so soon as the resultant pressure passes outside the middle third of the ring; the couple required to produce equilibrium would then require a negative force or tension at the opposite edge, and masonry cannot supply this tension, but in a metal rib the couple or bending moment produced by the excentricity of the stress may be resisted by the stiffness of the rib acting as a beam subject to a bending moment. Thus the strength of an arch to resist flexure is a more important element in the metal rib than in the masonry structure. It would be false to say that the ring of voussoirs had *no strength* to resist flexure, for we have on the contrary seen that the moment of the elastic forces at any section of a stone ring does resist any distorting action produced by the load; but in masonry this moment should never exceed the comparatively small value consistent with the absence of tension on any part of any joint. The metal rib may with safety be subjected to considerable tension in parts, and its strength to resist flexure can be easily increased and can be calculated with certainty. Moreover, by hinging the rib at one or both springings, as can be done with metal, the problem of determining the horizontal thrust (or total thrust) is simplified, the position of the thrust being thereby rendered certainly axial at this point, and then by taking into account the actual deformation of each part of the rib a complete solution of the problem of its strength can be obtained.

§ 46. *Horizontal Thrust of a Metal Arch or Rib hinged at the Abutments.*—By supporting a rib on pins or in cylindrical bearings (*vide* fig. 62) at the abutments we determine two points traversed by the thrust. The effect of allowing free rotation is necessarily to render the bending moment *nil* round the centre of rotation. Hence the resultant thrust must traverse the centre of the pin, or the centre of curvature of the bearing. Knowing the point of application of the thrust we have now to determine its magnitude. The vertical component *v* is the same as the load on the pier of a girder of the same span equally and similarly loaded, so that the problem reduces itself to the determination of *h* the horizontal component.

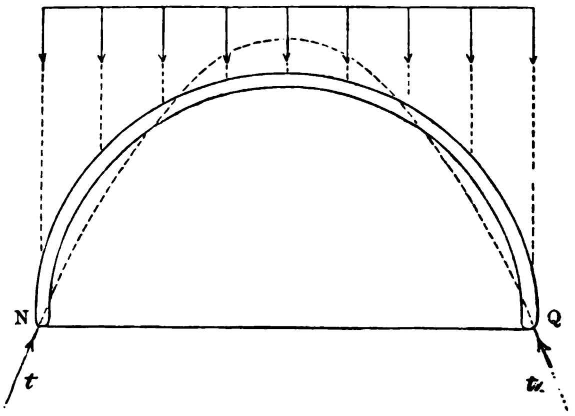


Fig. 57.

Let us first consider a semicircular rib (fig. 57), bearing a load uniformly distributed along the horizontal platform of the bridge (neglecting the weight of the rib). The linear arch will pass through the centre of the bearings N and Q, and will be a parabola. Moreover, it will be that parabola which requires the rib to exert no internal forces due to its own elasticity, and tending either to push out or draw in the springings; in other words, the rib, being supposed in equilibrium before the application of the weights, will not tend to act as a spring to increase or diminish the opening between N and Q. Nevertheless, as the semicircle cannot coincide with the parabola, most parts of the rib must be subject to bending moments, against which it will react as a bent spring. When the linear arch, as shown in fig. 57, passes above the axis of the rib at the crown, and below it at the haunches, the upper portion of the bent rib will act as a spring, tending by its reaction to diminish the distance between the ends N and Q, while the portions near the springing will be so bent as by their reaction to tend to increase that distance; now, if, as is necessarily the case, the whole rib is not to act as a spring, tending either to close or open the ends N and Q, then the effect of the bending near the haunches must exactly neutralize the effect of the bending near the crown. We have now to find what direction of thrust at the springing will give a linear arch such that the above condition may be fulfilled.

Let M be the bending moment acting at any given section, the centre or neutral axis of which is at a height y (fig. 58) above the horizontal line joining the springings; let this moment be considered constant for a short length ΔL of the rib measured axially along the rib; let Δs be the short distance measured horizontally by which the moment M acting throughout the length ΔL would increase or diminish the span of the rib at the springings, the rest of the rib being assumed free from strain. Then calling I the moment of inertia of the cross section of the rib, and E the modulus of elasticity, we shall have—

$$1. \quad . \; . \; . \; . \; . \; . \; . \quad \Delta s = \frac{My.\Delta L}{IE},$$

as appears from the following considerations:—

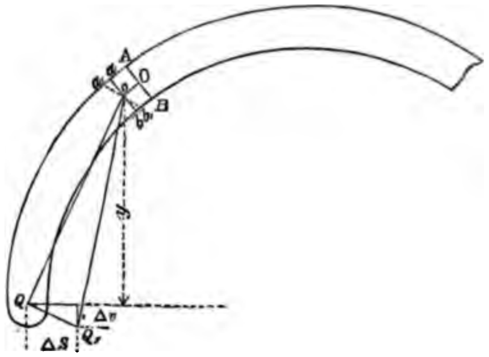


Fig. 58.

In fig. 58, let $Oo = Aa = Bb = \Delta L$; conceive the surface at AB as fixed, and let the action of the couple M be such as to extend the top and compress the bottom of the rib, moving the point a to a_1, and the points b to b_1; then, calling p the intensity of the stress at a, we have (equation 4, § 14)—

$$M = \frac{2p_1 I}{d};$$

also we have (§ 24)—

$$aa_1 = \frac{p_1 \Delta L}{E};$$

and therefore—

$$aa_1 = \frac{M\Delta L}{EI}\,\frac{d}{2};$$

join o and Q and draw the line oQ_1, making the angle QoQ_1 equal to aoa_1; at Q draw $\overline{QQ_1}$ perpendicular to $\overline{oQ}$. Then the effect of the couple M on the length ΔL of the rib, the rest being unstrained, will be to move the point Q to Q_1, and by similar triangles we have—

$$\overline{aa_1} : \overline{QQ_1} = \frac{d}{2} : \overline{oQ};$$

and therefore—

$$\overline{QQ_1} = \frac{M.\Delta L}{E.I}\,.\,\overline{oQ}\,.$$

Then resolving the motion $\overline{QQ_1}$ into horizontal and vertical components Δs and Δv, we have by similar triangles, $\Delta s : QQ_1 = y : oQ$, or, as above—

$$\Delta s = \frac{My.\Delta L}{I.E}.$$

But if, as above stated, the rib does not act as a spring in either direction, the span will remain constant, and the sum of all the changes in span produced by all the successive lengths ΔL will be *nil*, or $\Sigma\Delta s = 0$. Hence, since E is constant, we have

$$2. \quad . \; . \; . \; . \; . \; . \; . \quad \Sigma\,\frac{My.\Delta L}{I} = 0$$

as a necessary condition for the equilibrium of a loaded rib, hinged at the abutments when these do not yield. This condition must be satisfied whatever be the form or load of the rib, the reasoning by which it was obtained being independent of the form either of the rib or linear arch. When the cross section of the rib is constant, we have—

$$3. \quad . \; . \; . \; . \; . \; . \; . \quad \Sigma My.\Delta L = 0\,.$$

We shall now proceed to show how the linear arch satisfying this condition can be found for the case of a uniform rib.

Let the line $OO_1O_2O_3O_4$, fig. 59, be the geometrical axis of the rib, and let $OC_1C_2C_3C_4$ be the linear arch required; this arch will, as shown above, cut the geometrical axis at some point, as at O_3.

Let $\overline{C_4C}$ represent the resultant pressure on the rib at any point C_4 in direction and magnitude, fig. 60. If this pressure be resolved into its vertical and horizontal components, the latter $\overline{C_4H}$ will be equal to the horizontal thrust h (constant throughout the linear arch since the loads are vertical).

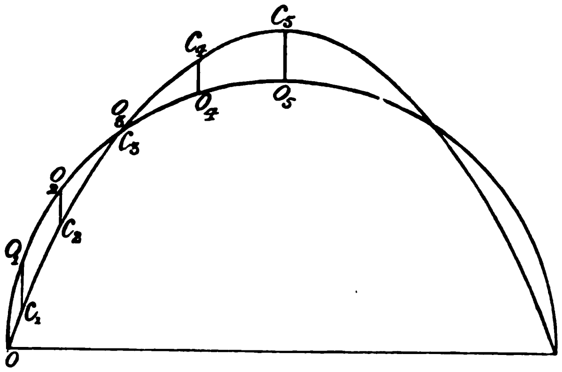


Fig. 59.

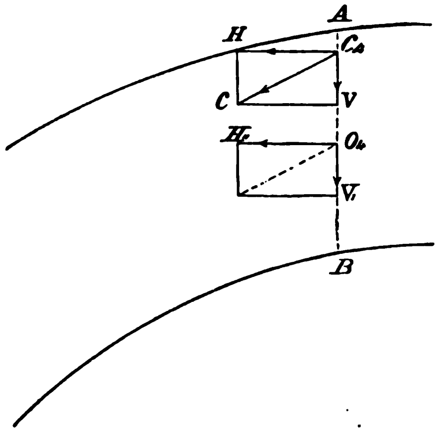


Fig. 60.

This force applied at C_4 will be equivalent to an equal and parallel force, $O_4H_{,,}$ applied at the point, O_4 in the axis, added to a left handed couple, of which the moment is $h\,\overline{O_4C_4}$. This couple is, for the point O_4, the couple M[*] required for equations 1, 2, and 3, the magnitude of which, h being constant, is proportional to the vertical distance OC between the curves at any point. Equation 3 requires that the sum of all the values of My shall be equal to zero, and we now see that this condition results when the sum of all the products of OC into y is equal to zero, or when—

$$4. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad \Sigma y\,.\,OC = 0\,;$$

when the cross section is not constant the value of $\frac{y.OC}{I}$ must be substituted for the simple product $y.OC$.

The problem of discovering the actual linear arch which will be called into play with a given rib is now reduced to that of finding the linear arch fulfilling the condition in equation 4. We might proceed tentatively, drawing numerous linear arches, and selecting by trial that which most nearly fulfils the condition, as proposed by Mr Bell, *Proc. I.C.E.*, vol. xxxii., but Professor Fuller of Belfast has shown, *Proc. I.C.E.*, vol. xl., that the ordinates of the required linear arch can at once be calculated from the values of the bending moment at the several sections of a beam of equal span and similarly loaded. Let $og_1g_2g_3$ (fig. 61), be the curve of bending moments which, as was shown in § 30, is one form of linear arch corresponding to the given load, the lengths $\overline{oo_1}$, $\overline{o_1o_2}$, $\overline{o_2o_3}$ &c., being equal and representing ΔL; let y as before be any ordinate of the curve $oo_1o_2o_3$, the axis of the rib; let $\overline{sg}$ be any ordinate of the given curve of bending moments; let $\overline{sc}$ be any ordinate of the required linear arch. Then, since $\overline{oc} = \overline{sc} - y$, we have for the case of uniform cross section and hinged abutments the equation—

$$4. \quad . \; . \; . \; . \; . \quad \Sigma y\,(\overline{sc} - y) = 0, \text{ or } \Sigma y.\overline{sc} = \Sigma y^2\,;$$

---

[*] This couple is sufficient to shift the force from C_4 to O_4, but the resultant of the force at C_4 and the couple would not be tangent to the geometrical axis of the rib. To alter the direction of the force in this manner a vertical component must be added, but this vertical component may be looked upon as a shearing force, which, being vertical, tends neither to extend nor to diminish the span.

but the ratio $\overline{sc} : \overline{sg}$ is constant, and may be designated by the letter k; so that we may write $k\,(\Sigma y . \overline{sg}) = \Sigma y^2$, from which equation we find the value of k—

$$5. \quad \ldots\ldots\ldots \quad k = \frac{\Sigma y^2}{\Sigma y . \overline{sg}} \,.$$

If the cross section is not constant we have for k the more complex expression—

$$6. \quad \ldots\ldots\ldots \quad k = \frac{\Sigma \dfrac{y^2}{I}}{\Sigma \dfrac{y . \overline{sg}}{I}} \,;$$

but $\overline{sc} = k . \overline{sg}$, so that the required ordinate $\overline{sc}$ is at once obtained in terms of k and the known ordinate $\overline{sg}$.

When the actual linear arch $oc_1c_2c_3$, &c., has been thus obtained, it is easy to calculate the horizontal thrust; for let $\overline{s_5c_5}$ be the maximum ordinate, the direction of the thrust will at this point of the curve be horizontal, and therefore calling W the weight on one side of this ordinate, and x the distance of its centre of gravity from the springing, we have—

$$6. \quad \ldots\ldots\ldots \quad Wx = h . \overline{s_5c_5} \,,$$

from which h can be found.

h and v being known give the position and direction of the resultant thrust at the springing of the rib. The magnitude of the thrust at any other point is easily computed graphically or by moments round the springing; then the resultant thrust at a given point being known, the intensity of the stress on any part of a section at that point is to be computed by first resolving the thrust into two components, one normal to the section and one in the plane of the section; the latter gives rise to a shearing stress (analogous to the force which causes one stone to slip on another in the masonry arch), while the component normal to the section will (if not axial) give rise to a uniformly varying stress, the magnitude of which at each distance from the axis can be computed by the formulæ given in § 8.

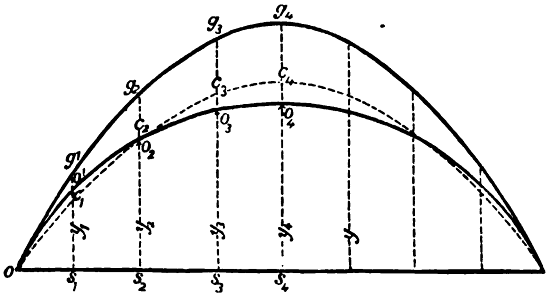


Fig. 61.

The value of h is determinate, if the direction of the rib be supposed fixed at the springing, but this cannot be ensured in large structures, and the theory need not therefore be developed. It simply requires $\Sigma\,\overline{co} = 0$.

When the rib (as is generally the case in existing bridges) abuts against a flat springing the exact value of h is indeterminate. When the rib is hinged the friction at the bearing renders the thrust indeterminate within limits depending on the possible bending moment at the springing due to the friction.

§ 47. *Process of Designing a Rib.*—In future designs of ribbed arches it is to be hoped that the practice will be adopted of allowing the rib freedom to turn at the springing. This can be done by ending the rib in a bearing, curved as in fig. 62; the resultant thrust will then be approximately axial, and the stress on every part of the rib can be determined with as much accuracy as on the several parts of a girder. When the span is large a cast-iron metal arch can be made lighter than a wrought iron girder for the same load, but the imperfection of the theory of the stresses on the ribs has hitherto led to great waste of metal in their construction. In what follows it is assumed that the resultant at the springing passes through the geometrical centre of the cross section of the rib.

If the rib were to carry a load distributed only in one way we ought clearly to make the form of the axis of the rib coincide with a linear arch for that load. There would then be no bending moment on any part of the rib. As in practice we must provide for all the possible combinations of passing load, we need take little pains in designing the curvature of the rib—a flat arc of a circle with a rise of say $\frac{1}{4}$th will answer well. The semicircular or elliptical forms are not good, for no linear arch with any practical distribution of load can even approximately coincide with a form in which the rib springs vertically from the abutments.

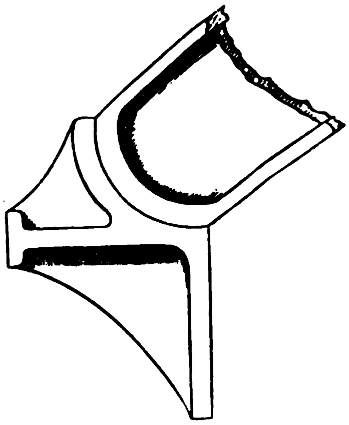


Fig. 62.

The general character of the cross section should be similar to that for a girder, inasmuch as the rib will have to resist bending moments as well as direct compression. The depth need not, however, be nearly so great as the depth of a girder. Let a cross section be chosen in which the area is assumed as approximately say 5 per cent. more than that which would be sufficient to sustain the thrust resulting from a linear arch suitable for the maximum load and coinciding approximately with the axis of the rib. (If the load be nearly uniform per foot run of platform we may for this first approximation take $h = \frac{wl^2}{8d}$, where d is the rise of the linear arch above the springings.) With the rib thus designed determine by the method given in § 46 the actual linear arches resulting from the following arrangements of passing load (combined with the permanent load):—(1), Bridge half covered from one end; (2), three-quarters covered from one end; (3), wholly covered; (4), covered by the passing load over the middle half of the bridge, the haunches being unloaded. Draw these linear arches on the rib by the method given in § 38, and choosing at each one of some eight or ten selected sections the two curves which most nearly approach the top and bottom flanges respectively, compute the maximum intensity of stress on the top and bottom flanges at each section from the two thrusts corresponding to these two linear arches; where the stress is excessive add metal; remove it where the maximum stress is less than the safe stress for the material. If no great change is made in the design this process will be sufficient, but if the cross section is seriously modified by the alteration we must make a second approximation by recalculating the linear arches for the new form of rib, and thus proceed by trial and error until the stresses corresponding to the actual linear arches are met by sufficient metal at all points. The rib need not be of uniform depth throughout, and may be increased in depth at the places where the stress due to bending moment has been found excessive.

In large spans the effect of a change of temperature must be taken into account. This can be done by finding the linear arch given by the expression—

$$1. \quad \ldots\ldots \quad \Sigma \frac{M \Delta l}{I} y = \Delta s \,,$$

where Δs is the alteration in span which would result from the expansion or contraction of the span if free to expand or contract with the change of temperature.

In a series of arches abutting against comparatively slender piers, account must be taken of the thrust transmitted from the neighbouring arch. This thrust will only be due to the passing load, and a part may be considered as taken by the pier; the remainder which the pier cannot counterbalance must be compounded with the reaction due to the linear arch calculated for the permanent load of the unloaded span. With the reaction thus computed the linear arch resulting in the unloaded span must be constructed, and the stresses examined on the top and bottom flanges of the rib.

The theory now given for a stiff rib used as an arch is equally applicable to a stiff rib hung as a suspension bridge.

§ 48. *Wooden Arches.*—Arches have occasionally been built of wood with ribs elaborately constructed of bent timber, scarfed and bolted together; the strength of such a rib could be calculated in the way indicated for metal ribs, but the mode of construction is not to be recommended. When wood is employed it should be used in simple straight balks built into a framed arch.

§ 49. *Practical Details of Metal Arches.*—The common form of metal arch is a cast-iron rib of I section and of small depth. This rib is intended to be sufficient, unaided, to bear the whole weight of the superstructure. The spandrils, made of some kind of lattice work (or occasionally a mere arcade), bear the roadway, and to some extent stiffen the rib beneath. The rib may with advantage be made much deeper than has been the practice, and may consist of tubes framed as in the St Louis Bridge, fig. 5, Plate XVIII., so as to form a single stiff rib. Where, to gain headway, a rib of small depth at the crown is desirable, the rib might with advantage be deepened at the haunches. Wrought iron is a very suitable material for small arches, where the permanent load is insufficient to prevent tension from occurring in some parts of the rib. Cast-iron and cast-steel are better materials for large spans; for moderate spans a good form of metal arch will be shown under the head of "Frames" (fig. 77), being that in which a lower member is braced to the upper member carrying the roadway so as to form a true frame; for very large spans a single deep rib, or a frame with parallel members arranged as an arch, may be adopted. This design has the advantage over that shown in fig. 77 of avoiding very long bracing at the abutments.

V. FRAMES.

§ 50. *Preliminary.*—A frame is a rigid structure composed of straight struts and ties. The struts and ties are called the members or pieces of the frame. The frame as a whole may be subject to a bending moment, but each bar, pillar, rod, or cord in the structure is thereby simply extended or compressed so that the total stress on a given member is the same at all its cross sections, while the intensity of stress is uniform for all the parts of any one cross section. This result must follow in any frame, the members of which are so connected that the joints offer little or no resistance to change in the relative angular position of the members. Thus if the members are pinned together, the joint consisting of a single circular pin, the centre of which lies in the axis of the piece, it is clear that the direction of the only stress which can be transmitted from pin to pin will coincide with this axis. The axis becomes, therefore, a line of resistance, and in reasoning of the stresses on frames we may treat the frame as consisting of simple straight lines from joint to joint. When the members of a frame consist of iron rods as ties, combined with struts formed by angle iron or T iron of the usual sizes, or by pieces of timber of the ordinary dimensions, it is found by experiment that the stresses on the several members do not differ sensibly whether these members are pinned together with a single pin or rigidly jointed by several bolts or rivets. Frames are much used as girders, and they also give useful designs for suspension and arched bridges. A frame used to support a weight is often called a *truss;* the stresses on the various members of a truss can be computed for any given load with greater accuracy than the intensity of stress on the various parts of a continuous structure such as a tubular girder, or the rib of an arch. Many assumptions are made in treating of the flexure of a continuous structure which are not strictly true; no assumption is made in determining the stresses on a frame, except that the joints are flexible, and that the frame shall be so stiff as not sensibly to alter in form under the load. Both assumptions are consistent with the facts in the case of any bridge truss.

§ 51. *Classes of Frames used as Trusses.*—Frames used as bridge trusses should never be designed so that the elongation or compression of one member can elongate or compress any other member. An example will serve to make the meaning of this limitation clearer. Let a frame consist of the five members AB, BD, DC, CA, CB (fig. 63), jointed at the points A, B, C, and D, and all capable of resisting tension and compression. This frame will be *rigid, i.e.,* it cannot be distorted without causing an alteration in the length of one or more of the members; but if from a change of temperature or any other cause one or all of the members change their length, this will not produce a stress on any member, but will merely cause a change in the form of the frame. Such a frame as this cannot be *self-strained.* A workman, for instance, cannot produce a stress on one member by making some other member of a wrong length. Any error of this kind will merely affect the form of the frame; if, however, another member be introduced between A and D, then if BC be shortened AD will be strained so as to extend it, and the four other members will be compressed; if CB is lengthened AD will thereby be compressed, and the four other members extended; if the workman does not make CB and AD of exactly the right length they and all the members will be permanently strained. These stresses will be unknown quantities, which the designer cannot take into account, and such a combination ought therefore never to be adopted. A frame of this second type is said to have one *redundant member.*

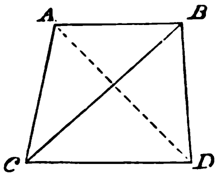


Fig. 63.

If the members AD and CB were flexible cords there would be no redundant members; for the tightening of one diagonal would throw no sensible stress on the other diagonal, since it is supposed incapable of resisting a thrust. Both diagonals, if flexible, are required to prevent the quadrilateral from getting out of shape. Members capable of bearing only one kind of strain might receive the name of *semi-members.*

§ 52. *External Forces on Frame.*—Frames used as bridge trusses are in equilibrium under the external forces applied to them. These forces are—(1) the loads, (2) the reactions at the points of support. The loads are to be referred to the joints as follows:—(1) find the resultant of the load carried by any two joints; (2) resolve that load into two vertical components acting through the two joints; (3) compound the several components acting at each joint into one resultant. This process gives a frame with external forces equivalent to the actual loads, but acting only at the joints. The frames are always supported at a joint, and the reactions of the supports are therefore also forces acting at joints. The load between any two joints is directly supported by the member of the frame joining them; the stresses due to the direct action of this partial load must, where great accuracy is wanted, be added to the stresses computed on the assumption that the loads have been applied directly to the joints. Generally the stresses due to the direct action of the load between two joints may be neglected except where a member of the frame is employed to carry the roadway.

Fig. 64 shows a common form of bridge truss known as

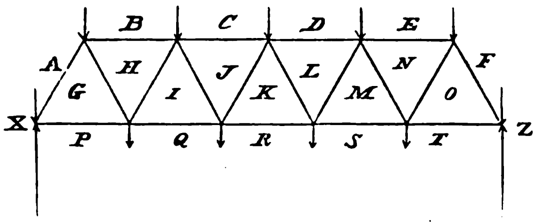


Fig. 64.

a *Warren girder*, with lines indicating external forces applied to the joints; half the load carried between the two lower joints next the piers on either side is directly carried by the abutments. The sum of the two upward vertical reactions must clearly be equal to the sum of the loads. The lines in the diagram represent the directions of a series of forces which must all be in equilibrium; these lines may, for an object to be explained in the next paragraph, be conveniently named by the letters in the spaces which they separate instead of by the method usually employed in geometry. Thus we shall call the first inclined line on the left hand the line AG, the line representing the first force on the top left hand joint AB, the first horizontal member at the top left hand the line BH, &c.; similarly each point requires at least three letters to denote it; the top first left hand joint may be called ABHG, being the point where these four spaces meet. In this method of lettering, every enclosed space must be designated by a letter; all external forces must be represented by lines *outside* the frame, and each space between any two forces must receive a distinctive letter; this method of lettering was first proposed by Mr R. H. Bow (*Economics of Construction*), and is convenient in applying the theory of reciprocal figures to the computation of stresses on frames.

When the weight of the truss is small it is usual to refer the weights of the parts of the truss itself to the same joints as carry the roadway, and to treat all other joints as unloaded.

The reactions at the points of supports of a framed arch or suspension bridge are inclined, as in fig. 65; the manner of computing the stresses on these frames when the direction of these reactions is known will be first explained, and subsequently the manner of finding this direction will be given.

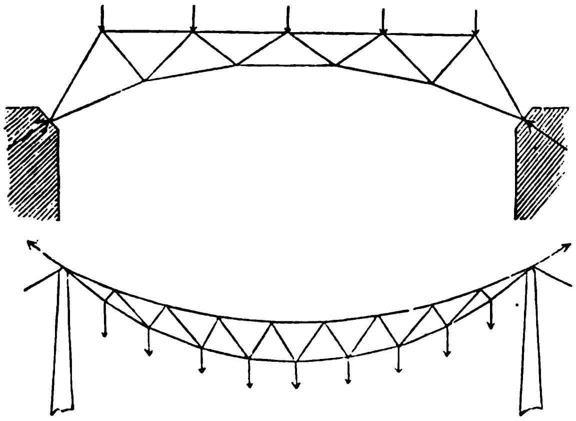


Fig. 65.

§ 53. *Reciprocal Figures.*—Prof. Clerk Maxwell has given (*Phil. Mag.* 1864), the following definition of reciprocal figures:—"Two plane figures are reciprocal when they consist of an equal number of lines so that corresponding lines in the two figures are parallel, and corresponding lines which converge to a point in one figure form a closed polygon in the other."

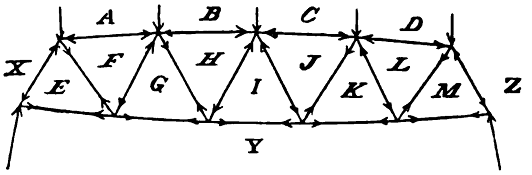
Let a frame (without redundant members), and the external forces which keep it in equilibrium, be represented by a diagram constituting one of these two plane figures, then the lines in the other plane figure or the reciprocal will represent in direction and magnitude the forces between the joints of the frame, and, consequently, the stress on each member as will now be explained.

Reciprocal figures are easily drawn by following definite rules, and afford therefore a simple method of computing the stresses on members of a frame.

The external forces on a frame or bridge in equilibrium under those forces may, by a well-known proposition in statics, be represented by a closed polygon, each side of which is parallel to one force, and represents the force in magnitude as well as in direction. The sides of the polygon may be arranged in any order, provided care is taken so to draw them that in passing round the polygon in one direction this direction may for each side correspond to the direction of the force which it represents.

This polygon of forces may, by a slight extension of the above definition, be called the *reciprocal figure* of the external forces, if the sides are arranged in the same order as that of the joints on which they act, so that if the joints and forces be numbered 1, 2, 3, 4, &c., passing round the outside of the frame in one direction, and returning at last to joint 1, then in the polygon the side representing the force 2 will be next the side representing the force 1, and will be followed by the side representing the force 3, and so forth. This polygon falls under the definition of a reciprocal figure given by Clerk Maxwell, if we consider the frame as a point in equilibrium under the external forces.

Fig. 66.



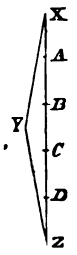


Fig. 66*a*.

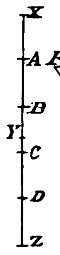


Fig. 66*b*.

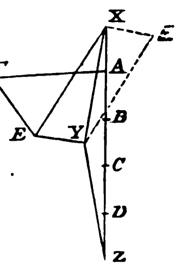


Fig. 66*c*.

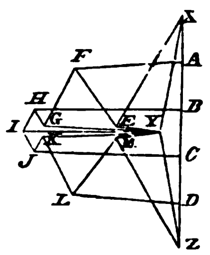


Fig. 66*d*.

Fig. 66 shows a frame supported at the two end joints, and loaded at each top joint. The loads and the supporting forces are indicated by arrows. Fig. 66*a* shows the reciprocal figure or polygon for the external forces on the assumption that the reactions are slightly inclined. The lines in fig. 66*a*, lettered in the usual manner, correspond to the forces indicated by arrows in fig. 66, and lettered according to Mr Bow's method. When all the forces are vertical, as will be the case in girders, the polygon of external forces will be reduced to two straight lines, fig. 66*b*, superimposed and divided so that the length AX represents the load AX, the length AB the load AB, the length YX the reaction YX, and so forth. The line XZ consists of a series of lengths, as XA, AB......DZ, representing the loads taken in their order. In subsequent diagrams the two reaction lines will, for the sake of clearness, be drawn as if slightly inclined to the vertical (as practised by Mr Bow).

If there are no redundant members in the frame, there will be only two members abutting at the point of support, for these two members will be sufficient to balance the reaction, whatever its direction may be; we can therefore draw two triangles, each having as one side the reaction YX, and having the two other sides parallel to these two members; each of these triangles will represent a polygon of forces in equilibrium at the point of support. Of these two triangles, shown in fig. 66*c*, select that in which the letters X and Y are so placed that (naming the apex of the triangle E) the lines XE and YE are the lines parallel to the two members of the same name in the frame (fig. 66). Then the triangle YXE is the reciprocal figure of the three lines YX, XE, EY in the frame, and represents the three forces in equilibrium at the point YXE of the frame. The direction of YX, being a thrust upwards, shows the direction in which we must go round the triangle YXE to find the direction of the two other forces; doing this we find that the force XE must act down towards the point YXE, and the force EY away from the same point. Putting arrows on the frame diagram to indicate the direction of the forces, we see that the member EY must pull and therefore act as a tie, and that the member XE must push and act as a strut. Passing to the point XEFA we find two known forces, the load XA acting downwards, and a push from the strut XE, which, being in compression, must push at both ends, as indicated by the arrow, fig. 66. The directions and magnitudes of these two forces are already drawn (fig. 66a) in a fitting position to represent part of the polygon of forces at XEFA; beginning with the upward thrust EX, continuing down XA, and drawing AF parallel to AF in the frame we complete the polygon by drawing EF parallel to EF in the frame. The point F is determined by the intersection of the two lines, one beginning at A, and the other at E. We then have the polygon of forces EXAF, the reciprocal figure of the lines meeting at that point in the frame, and representing the forces at the point EXAF; the direction of the forces on EX and XA being known determines the direction of the forces due to the elastic reaction of the members AF and EF, showing AF to push as a strut, while EF is a tie. We have been guided in the selection of the particular quadrilateral adopted by the rule of arranging the order of the sides so that the same letters indicate corresponding sides in the diagram of the frame and its reciprocal. Continuing the construction of the diagram in the same way, we arrive at fig. 66d as the complete reciprocal figure of the frame and forces upon it, and we see that each line in the reciprocal figure measures the stress on the corresponding member in the frame, and that the polygon of forces acting at any point, as IJKY, in the frame is represented by a polygon of the same name in the reciprocal figure. The direction of the force in each member is easily ascertained by proceeding in the manner above described. A single known force in a polygon determines the direction of all the others, as these must all correspond with arrows pointing the same way round the polygon. Let the arrows be placed on the frame round each joint, and so as to indicate the direction of each force on that joint; then when two arrows point to one another on the same piece, that piece is a tie; when they point from one another the piece is a strut. It is hardly necessary to say that the forces exerted by the two ends of any one member must be equal and opposite. This method is universally applicable where there are no redundant members. The reciprocal figure for any loaded frame is a complete formula for the stress on every member of a frame of that particular class with loads on given joints. Some examples of these figures will be given, and the reader will easily construct others for himself.

§ 54. *Warren Girders—Reciprocal Figure and Method of Computing Stresses by Method of Sections.*

Case 1. The Warren girder loaded at each top joint, figs. 67 and 67*a*. This diagram differs very slightly from that shown in fig. 66. The top and bottom members are in straight lines, and consequently the lines indicating the stresses on the bottom member are superimposed one on the other instead of radiating from Y; the loads XA and FZ are shown as directly borne by the piers. It is clear that if the road is supported by a platform reaching from the end joints to the piers, half of the load on these parts of the platform will be directly supported on the piers. These end loads are shown also in the subsequent diagrams. The truss is generally built of equilateral triangles, and the inclination of the bracing to the horizon should never be less than 45°.

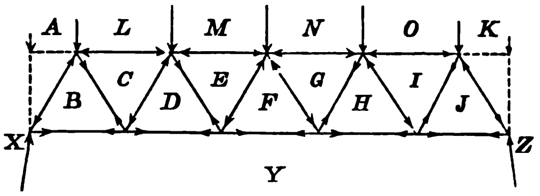


Fig. 67.

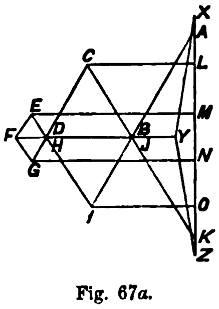


Fig. 67*a*.

Case 2. Warren girder loaded on top and bottom, fig. 68. 68*b* shows the polygon of external forces, and 68*c* shows half the reciprocal figure. These figures have been added to facilitate the comprehension of the complete reciprocal shown by 68*a*.

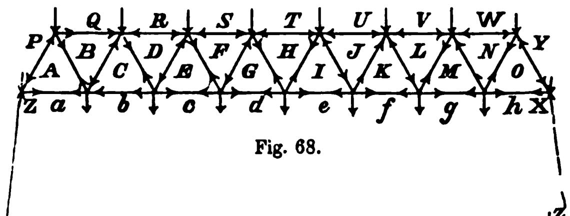


Fig. 68.

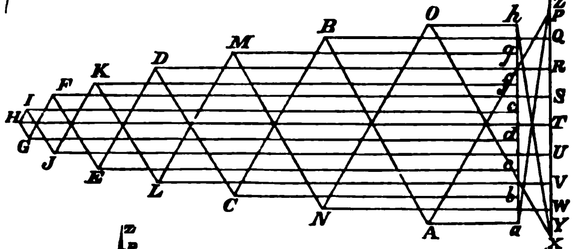


Fig. 68*a*.

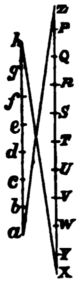


Fig. 68*b*.

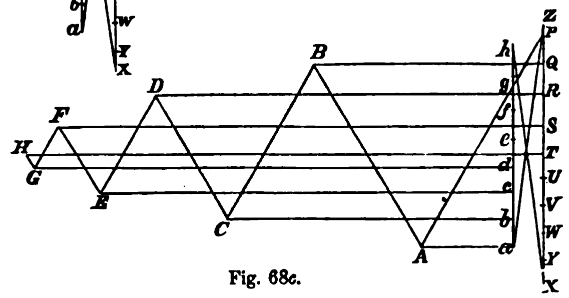


Fig. 68*c*.

Case 3. Warren girder with one load, not central, fig. 69. The polygon RXZ, fig. 69*a*, represents the external forces.

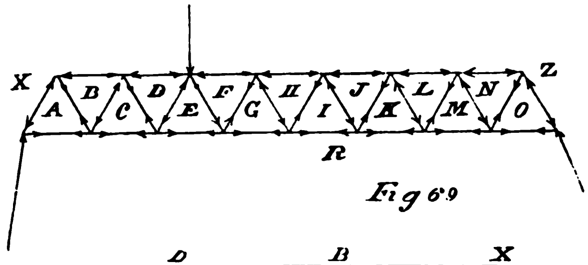


Fig 69

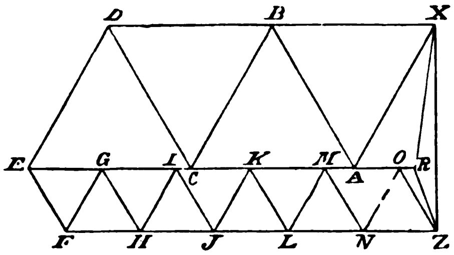


Fig. 69*a*.

Case 4. Warren girder; load on half the top joints, fig. 70.

In designing a Warren girder it is necessary to provide for the advancing load. With a uniformly distributed load the diagonal bracing consists of alternate struts and ties beginning, if supported at the bottom, with a strut at each end, or with the tie at each end if the truss is hung from the top. At the centre there will be either two ties or two struts in juxtaposition; or, as in the case where there are an even number of loaded joints at the top, the central pair of diagonal members will be unstrained; but an advancing load (neglecting the permanent load) will convert each diagonal member (except the end ones) alternately into a strut and a tie. It is necessary to provide for the maximum extension and maximum compression on each, taking into account the combined effects of the permanent and passing load; the latter has generally sufficient effect to reverse the direction of the stress on the bracing near the centre, but not (in large bridges) towards the ends. Each member of the bracing towards the centre must, therefore, be so designed as to be capable of acting alternately as a strut and as a tie. The maximum stress on the top and bottom members occurs when the bridge is fully loaded.

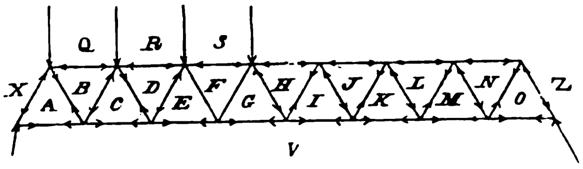


Fig. 70.

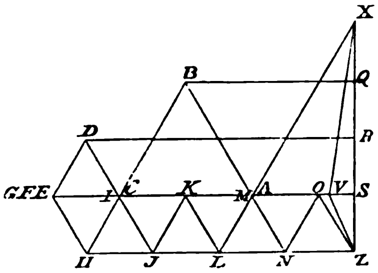


Fig. 70*a*.

This stress is easily calculated by the method of sections; assume the girder cut by a vertical plane at the joint or pin opposite the member in question, or in other words, by a vertical plane passing through the vertex of the triangle of which the member in question is the base; let d be the perpendicular distance between the member and the pin, t the thrust or tension on the member, and M the bending moment for the section calculated as for a girder loaded at the points corresponding to the joints. Then $M = td$. The stress on the diagonals corresponds to the shearing stress in the solid girder, and may consequently be calculated from the shearing diagrams, examples of which were given in § 19. The loads may be referred to the joints before drawing the diagram, and the continuous curves of figures 18 to 18*c* will be replaced by lines consisting of a series of steps such as are shown in fig. 18*a*. We may then proceed to calculate the stress on any diagonal as follows:—Take the shearing stress for the section at the joint where the diagonal in question abuts; if there is a load on this joint, take the shearing stress for a section close to the joint, and on that side of it which is next the diagonal in question; call the shearing force thus found F; let i be the slope of the diagonal or the angle which it makes with the horizon, then the tension or compression on the diagonal is F cosec. i; thus let fig. 71 represent a Warren girder of six equal bays, in which $i = 60°$. Let the load on each top joint be 5 tons, the compressions and tensions on the diagonals are as follows:—

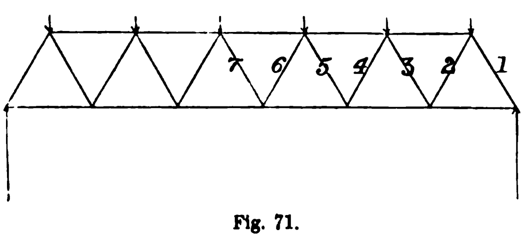


Fig. 71.

TABLE XI.

Name of Brace.	F × cosec. i.	Compression.	Tension
1	15 × 1·1547	17·32	...
2	10 × 1·1547	...	11·55
3	10 × 1·1547	11·55	..
4	5 × 1·1547	...	5·77
5	5 × 1·1547	5·77	...
6	0 × 1·1547	...	...

These stresses, given simply as an example, apply to the one special load, and are not to be confounded with the maximum stresses which an advancing load of the same intensity would produce. The stresses on the two halves of the girder are symmetrical.

The arithmetical mode of computation is the simpler where the top and bottom members are parallel and the inclination of the diagonals constant; where these conditions are not fulfilled the method by reciprocal figures is preferable.

In the actual design of any girder to suit various combinations of loads, care must be taken to design each member to suit the *maximum* stress which can arise from any combination. The maximum shearing stress is most easily selected by means of the diagrams § 19. Care must be taken to meet both the maximum tension and maximum compression whenever the member is so placed that with some loads it is extended and with some compressed. We have just shown that this case arises in the diagonals near the centre of the girder. When frames are used as continuous girders, it is desirable to make the points of inflection coincide exactly with a joint. This may be done by cutting through or omitting the member opposite the joint. This allows the reactions on each pier to be easily determined by the elementary principles of statics with any load, and without taking into account the form which the beam assumes when deflected. Instead of a long continuous girder, we then have a series of girders, supported at or near the middle of their length by the piers, while a second series hang from the first by pins at determinate points. This arrangement greatly simplifies all the calculations without sensibly diminishing the advantage derived from the use of continuous girders.

§ 55. *Various Forms of Girder.*—The framed girder is sometimes made of the form in fig. 72, which has the advantage of reducing the length of the end diagonals where the stress is heaviest. The reciprocal figure for this truss uniformly loaded on the bottom joints is shown in fig. 72*a*. This girder is sometimes called a bowstring girder, though this name more properly belongs to an obsolete form with no diagonals. Z and X are the spaces between the end loads and the reactions which, for clearness, are shown as pulling up; but the same reciprocal would result if the reaction were shown pushing up, and the letters X and Z were placed as dotted. This form has the advantage of reducing the compression on the diagonal struts to a comparatively small amount. A great part of the shearing stress is taken by the curved upper boom; deeper girders can be profitably used of the bowstring than of the Warren type. The long and expensive struts in the latter form more than counterbalance

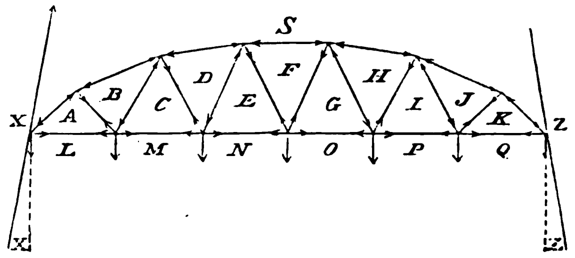


Fig. 72.

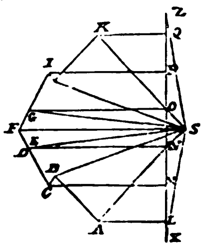


Fig. 72*a*.

the saving to be obtained by increasing the depth beyond about $\frac{1}{12}$th of the span.

The diagonal bracing in the Warren and bowstring girders is sometimes arranged as in fig. 73; the girder is then called a lattice-girder. It is to be treated as a series of superposed Warren girders, each bearing its share of the load. This form has a slight advantage inasmuch as the diagonal struts are stiffened by being pinned to the ties where they cross.

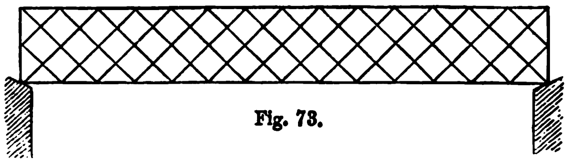


Fig. 73.

Fig. 74 shows a very common and useful form of girder

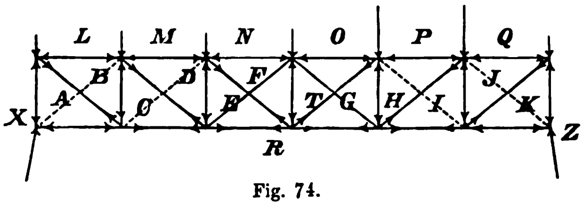


Fig. 74.

where the uprights between the top and bottom members are able to sustain both tension and compression, while the diagonals are only "semi-members," being flexible rods or bars. The reciprocal for this design is shown in fig. 74*a* with two joints more heavily loaded than the others. The members in fig. 74 which have no arrows on them are idle or unused, with this particular distribution of load. When the permanent load is considerable as compared with the passing load, the end or two end diagonals shown dotted will not come into action with any distribution of load, and may therefore be omitted. This form is much used when wood is employed for the compression members. It has in every case the advantage that the struts in the bracing are shorter than in the Warren girder. The lettering is arranged so that only those spaces have letters which are divided by members actually in use under this particular load.

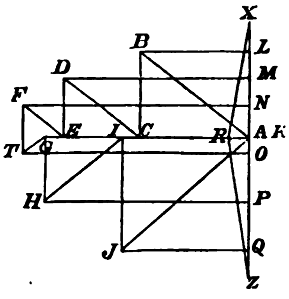


Fig. 74*a*.

Figs. 75 and 75*a* show a modification of this truss for small spans, and its reciprocal when loaded on one joint; the lettering here also only suits the one distribution of load, and the idle members have no arrows on them.

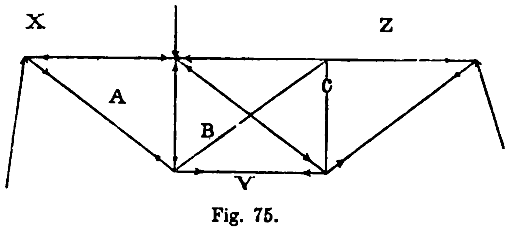


Fig. 75.

§ 56. *Framed Suspension Bridges and Arches.*—These frames, like the girders, consist of top and bottom members, braced together by ties and struts. The bridge is a suspension bridge, if the frame is supported by inclined forces pulling outwards from the bridge as in fig. 76, and an arch is supported by forces pushing inwards as in fig. 77. The reciprocals of these two forms with the joints at the platform uniformly loaded are annexed. These reciprocals are drawn on the hypothesis that the direction of the thrust or pull is known; and this has been chosen in this case so as to reduce the stress on NG to zero, as would necessarily be the case if NG were omitted or cut. When this is not the case the direction of the thrust or pull at the abutments must be found in the manner explained below.

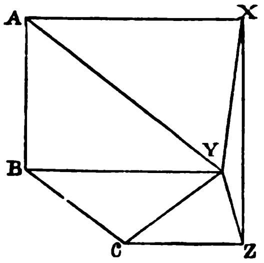


Fig. 75*a*.

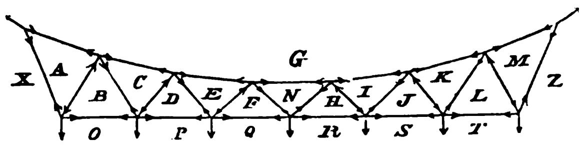


Fig. 76.

It is easy to design the bridge so that both the top and bottom members of the suspension bridge remain in tension, and both those of the arch in compression under all distributions of load. This would allow wire rope to be used for both members of the suspension bridge, and cast-iron or steel for both members of the arch. The stresses on the bracing are very uniform and small as compared with those on the diagonals of a girder.

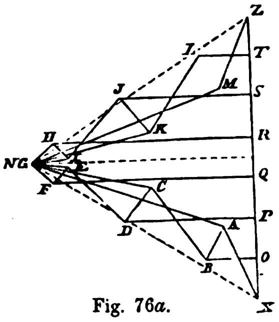


Fig. 76*a*.

Fig. 78 shows a slight modification of the design for a suspension bridge, very suitable for spans so large that the end struts in the preceding form would be inconveniently long. The reciprocal annexed is drawn for the case in which a double load is placed on half the bridge. The same design is suitable for an arch.

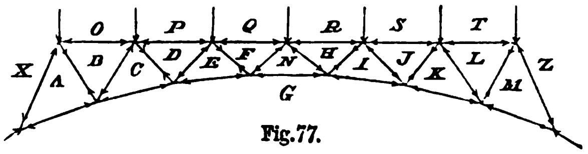


Fig. 77.

The resultant thrust or tension at the supports of framed suspension bridges or arches can only be found by a method analogous to that already explained for the solid metal rib. This method was first given by Prof. Clerk Maxwell.

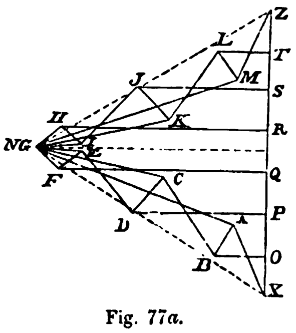


Fig. 77*a*.

Consider any member A, fig. 79, of length L and cross-section *a*, made of a material having the modulus of elasticity E; under the action of a stress F, the length L will be altered by an amount—

$$\Delta L = F \frac{L}{E.a}.$$

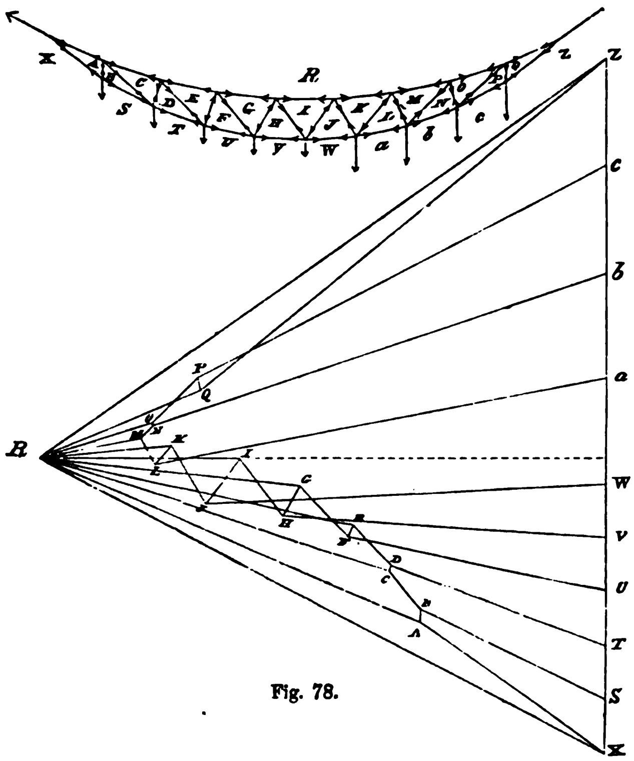


Fig. 78.

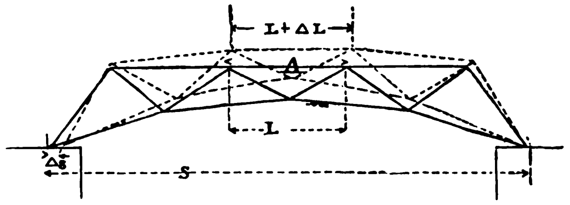


Fig. 79.

Appropriate signs must be given to the arithmetical values of the force and the alteration of length; thus thrust and compression may be called negative, pull and extension positive. If every other member of the frame were absolutely rigid, the span S of the bridge would undergo an alteration ΔS. The ratio $\frac{\Delta S}{\Delta L}$ will depend merely on the geometrical form of the frame; let q be the value of this constant ratio, so that—

$$1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \Delta S = q \Delta L = qF \frac{L}{E.a} .$$

Let f be the force produced on the member A by a horizontal force h acting between the springings; then by the principle of virtual velocities we have—

$$f : h = \Delta S : \Delta L,$$

for we may consider the structure merely as a kind of lever, by which the force h exerts a force on A, the fulcrum being the joint opposite A; then the above proportion expresses the fact that the forces h and f are inversely proportional to the spaces which the ends of the lever would move through when a small displacement occurs. Thus we may write

$$f = qh .$$

Similarly let f_1 be the stress which would be produced on A by a vertical force v, applied at one springing, while the other springing was held rigidly so that the whole frame could not turn; the ratio between f_1 and v is constant, depending merely on the form of the frame, so that we may write—

$$f_1 = pv ,$$

where p is a constant, to be found in the same manner as q; p may be defined as equal to the whole stress produced on the given member by a unit *vertical* force at the springing, and q as the whole stress produced on the same member by a unit *horizontal* force at the springing, the frame being held rigidly at the opposite abutment. Then any thrust t at the springing having the vertical and horizontal components v and h will produce a stress F on A, equal to the sum of the stresses of f and f_1, or—

$$F = qh + pv .$$

Substituting this value of F in equation 1 we have—

$$2. \quad . \quad . \quad . \quad \Delta S = (q^2 h + pqv) \left(\frac{L}{Ea}\right) = eq^2 h + epqv ,$$

where $e = \frac{L}{Ea}$, a constant for each member.

Now, let the values of q, p, and e be calculated for every member of the frame; then, calling the whole elongation $\Sigma \Delta S = k$, we have—

$$k = \Sigma (eq^2 h + epqv) .$$

If the abutments do not yield $k = 0$, and for this case—

$$3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad h = \frac{\Sigma epqv}{\Sigma eq^2} .$$

v is to be found as for a girder similarly loaded, and t, the required thrust or tension, is the resultant of h and v. The calculation is best made as follows:—Construct tables of the values of d and q for each member of the frame; the method of sections or moments will answer best for the top and bottom members, and that of reciprocal figures for the diagonals; assume a cross section for each member, based on a probable assumed value of t; for the required load make tables of epq and eq^2, or what is equivalent, when E is constant, make tables of the values of $\frac{pq^2}{a}$ and $\frac{q^2 L}{a}$. The sum of eq^2 or $\frac{q^2 L}{a}$ can then be made. If there be a load on one joint only, find the values of v and v_1, for the right and left abutments, then find $\Sigma epqv$, using the value v for all members to the right of the load, and v_1 for all members to the left of the load; equation 3 will now give the value of h for this single load.

The process of finding $\Sigma epqv$ must be repeated for each joint which is loaded, and the whole horizontal thrust due to the load on any number of joints will be the sum of the separate values of h. When the horizontal thrust is known, the thrusts t and t_1 are obtained at the two abutments by compounding the horizontal thrust with the vertical weight borne at that abutment. When t and t_1 are known, the stresses on each member are to be computed by reciprocal figures or any other convenient method. The process must be repeated for each combination of passing and permanent load, so as to find the maximum stress to which any member can be subjected. If the assumed cross sections are not suitable for these stresses, fresh cross sections must be assumed and the whole calculation repeated. The change in cross section will cause some change in the values of h, but this tentative process need seldom be gone through more than once.

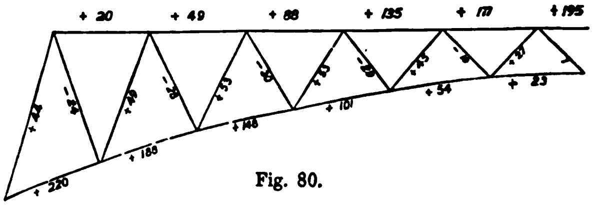


Fig. 80.

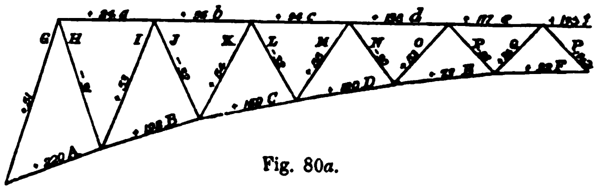


Fig. 80*a*.

Fig. 80 shows half the frame of a bridge for which the calculations have been made. The span is 120 feet, the rise 12 feet, and the truss 5 feet deep at the crown. The load assumed is 10 tons permanently on each top joint, and 10 tons of passing load. On fig. 80 are marked the stresses when the bridge is wholly covered with the passing load. On fig. 80*a* are marked the maximum stresses with any distribution of load. Figs. 81 and 82 show graphi-

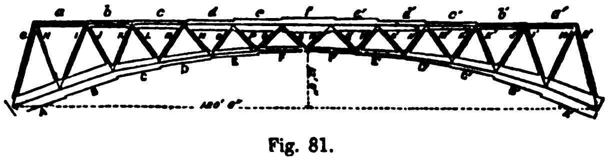


Fig. 81.

cally the amount of metal required in this bridge and in a girder of the same span and 12 feet deep; the breadth given in the diagram to each member is proportional to the cross section required. The quantity of metal required for the girder exceeds that for the arch in the ratio of about 175 to 100; a similar calculation for a bowstring lattice girder 17 feet deep at the centre gives the ratio between the weights of metal required as 100 for the arch and 155 for the bowstring. Even these ratios understate the great advantage to be derived from the braced arch or suspension bridge for large spans, since they assume that the loads to be carried are the same, whereas the permanent load is in large spans much less for the lighter construction of bridge. The following table shows the probable weight in tons of several different types of trusses, assuming that the maximum intensity of stress on the metal in girders and arches is everywhere 4 tons per square inch, that the passing load to be carried is 1·4 tons per foot run, and that the practical weight is 25 per cent. in excess of the minimum possible weight if no metal were wasted, vide *Trans. R.S.S.A.*, vol. viii. p. 135. It must be remembered that the abutments for arches will in all cases be more expensive than those for girders; in small spans this expense will often outweigh the saving which could be effected in the superstructure by employing the arch.

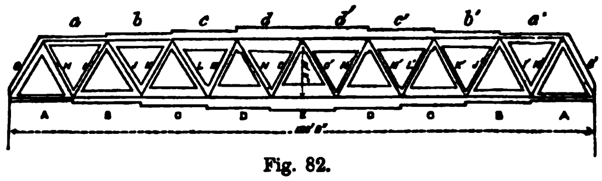


Fig. 82.

TABLE XII.—*Weights of Trusses of different types.*

Span.	Weight of wrought iron Girder in tons.		Weight of wrought iron Bowstring, in tons.		Weight of wrought iron Suspension Bridge, and wrought iron or cast iron Arch, in tons.		Weight of wooden Arch, in tons.		Weight of Suspension Bridge, in tons. Strain, 8 tons per square inch.	
1. Feet.	2. Total.	3. Per ft. run.	4. Total.	5. Per foot run.	6. Total.	7. Per ft. run.	8. Total.	9. Per ft. run.	10. Total.	11. Per ft. run.
200	83	·415	70	·35	41	·205	57·1	·285	19	0·095
300	222	·74	180	·60	100	·33	142	·473	45	0·15
400	475	1·19	375	·94	194	·485	285	·712	83	0·207
500	940	1·88	700	1·4	330	·65	510	1·02	134	0·268
600	2,220	3·7	1260	2·1	530	·883	870	1·45	200	0·333
700	3,900	5·57	2290	3·27	790	1·13	1440	2·06	280	0·400
800	12,800	16·	4450	5·56	1180	1·475			384	0·48
1000					2490	2·49			655	0·655
1200					5600	4·67			1060	0·885
1400									1580	1·128
1600									2360	1·475
1800									3410	1·89
2000									4950	2·475

The framed arch is a very suitable form for wooden bridges. The ties are few and subject to insignificant strains. In designing a series of arches supported on piers care must be taken to provide for the thrust from loaded to unloaded arches across the pier at the springing.

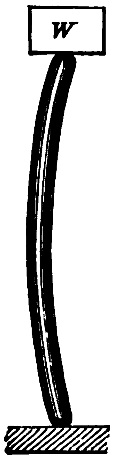


Fig. 83.

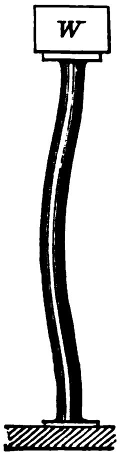


Fig. 84.

§ 57. *Strength of Struts.*—When a strut or column is used as in framework to resist compression, it is usually so long in comparison with its cross section that it will bend and yield with a much less stress than would be required to crush the material. The strength of a strut of this kind can be approximately computed according to the following theory:—

Let a strut with a cross section S be pinned or hinged at both ends, or let it have round ends, so that if it yields under compression it bends as in fig. 83. Let the cross section have two axes of symmetry, and let the section be such that the column will bend in the plane of one axis; let the depth of cross section measured in this direction be called d, and the breadth measured at right angles to the depth d be called b. Let the maximum or breaking load be called P, and the maximum deflection of the longitudinal axis of the strut from its unbent position be called v, this quantity being analogous to the deflection in a beam. The moment tending to produce flexure at the cross section where v is measured will be Pv. This moment must, as in the case of a girder, be equal to the moment of the elastic forces, which we already know to be—

$$u = \frac{2p_1 \mathrm{I}}{d},$$

where p_1 and I have the same signification as for girders.

(*I* must be taken about the unstrained axis, or in other words about the axis running in the direction which has been called that of the breadth.) Hence we have—

$$\mathrm{P}v = \frac{2p_1 \mathrm{I}}{d}, \text{ or } p_1 = \frac{\mathrm{P}vd}{2\mathrm{I}};$$

but we also know that for beams of uniform cross section under similar internal stresses v is proportional to $\frac{l^2}{\mathrm{I}}$; hence we have—

$$1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad p_1 = a\frac{\mathrm{P}l^2}{\mathrm{I}},$$

where a is a constant depending on the material only. Let $p_0 = \frac{\mathrm{P}}{\mathrm{S}}$ be the mean intensity of stress which would be produced if the load compressed all parts of the cross section equally, and let f_e as before be the ultimate strength of the material per unit of cross section, then, when the beam is on the point of yielding, we must have $f_e = v_0 + p_1$, or—

$$f_e = \frac{\mathrm{P}}{\mathrm{S}} + a\frac{\mathrm{P}l^2}{\mathrm{I}},$$

and calling r the radius of gyration—

$$2. \quad . \quad . \quad . \quad . \quad f_e = \frac{\mathrm{P}}{\mathrm{S}}\left(1 + a\frac{l^2}{r^2}\right), \text{ or } \mathrm{P} = \frac{f_e \mathrm{S}}{1 + \frac{al^2}{r^2}},$$

from which f_e can be computed in terms of P and a, or P in terms of f_e and a.

The value of a does not change with a change in the design of the cross section; the values for cast iron, wrought iron, and wood determined by experiment are given in the following table, S being measured in square inches, P in lbs. The table also contains values of f_e.

TABLE XIII.—*Constants for the Strength of Struts and Pillars.*

	f_e.	a.
Cast-iron................	80,000 lbs.	$\frac{1}{6400}$
Wrought iron	36,000 lbs.	$\frac{1}{36000}$
Dry timber	7,200 lbs.	$\frac{1}{3000}$

When the direction of both ends of the column is fixed it must bend in the manner shown in fig. 84; if the curvature be uniform this would have the effect of reducing the deflection v to one-fourth of the amount caused by the same stresses when the ends are hinged, giving—

$$p_1 = \frac{a\mathrm{P}l^2}{4\mathrm{I}},$$

and

$$3. \quad . \quad . \quad . \quad f_e = \frac{\mathrm{P}}{\mathrm{S}}\left(1 + \frac{al^2}{4r^2}\right), \text{ or } \mathrm{P} = \frac{f_e\mathrm{S}}{1 + \frac{al^2}{4r^2}}.$$

When one end is fixed in direction and the other hinged, the ultimate load which the strut can bear may be taken as a mean between the strength of two pillars of the same length and cross section, one having both ends fixed in direction and the other having both ends hinged; for similar cross sections r^2 is proportional to d^2, the square of the depth (measured in the direction of the shorter side or axis), thus the ratio $\frac{l^2}{r^2} \propto \frac{l^2}{d^2}$.

Let $n = \frac{d^2}{12r^2}$, and let $\mathrm{B} = 3\frac{al^2}{d^2}$. Then from equation 2 we have for hinged ends—

$$4. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \mathrm{P} = \frac{f\mathrm{S}}{1 + 4n\mathrm{B}},$$

and from equation 3 for fixed ends—

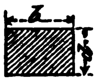
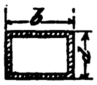
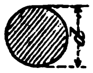
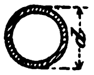
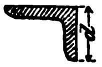
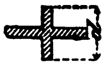
$$5. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \mathrm{P} = \frac{f\mathrm{S}}{1 + n\mathrm{B}}.$$

n has a constant value for all similar cross sections, and B a constant value for any given materials and any constant ratio of l to v. Table XIV. gives the values of B for a series of these ratios, and Table XV. gives the values of n for the most usual cross sections.

TABLE XIV.—*Value of* B (*lbs. and square inches*), *Strength of Struts and Pillars.*

$\frac{l}{d}$	B for Cast Iron.	B for Wrought Iron.	B for Strong Dry Timber.
10	·187	0·033	·25
15	·42	·075	·9
20	·75	·133	1·6
25	1·16	·208	2·5
30	1·69	·3	3·6
35	2·3	·408	4·9
40	3·	·533	6·4
45	4·68	·833	10·

TABLE XV.—*Values of* n, *Strength of Struts and Pillars.*

	n	
Square or side d, or rectangle with smallest side d..............	1	
Hollow rectangle, thin sides	½	
Circle, diameter d	1⅓	
Thin ring, external diameter d	⅔	
Angle iron, smallest side d...........	2	
Cruciform, smallest breadth $=d$......	2	

It is of great importance that the connections between the several struts and ties forming a frame should be so designed that the stresses produced may be *axial*. If this is not done the maximum intensity of stress on the strut or tie may greatly exceed those computed on the principles explained in the present paragraph (*vide* § 8).

Mr Unwin in his lectures gives the following empirical rules for the strength of wrought iron struts:—

$$p_1=\frac{fS}{1+\frac{l^2}{ch^2}} \text{ for fixed ends, } \quad . \quad . \quad . \quad . \quad . \quad 6.$$

$$p_1=\frac{fS}{1+\frac{4l^2}{ch^2}} \text{ for round ends, } \quad . \quad . \quad . \quad . \quad 7.$$

where different values are given to f and c according to the different cross sections of the struts.

Rectangular bars $c=2500$ $f=17$ tons.
Cylindrical bars $c=3500$ $f=17·5$,,
Angle, T, cross, and charcoal iron $c=900$ $f=19$,,

The following are Mr Hodgkinson's formulæ for the strength of cast-iron cylindrical pillars, the length of which is not less than thirty times the diameter:—Let P be the load which will produce failure, d the external diameter in *inches*, L the length in *feet*, and A a constant multiplier; then for solid pillars with either round or flat ends—

$$8. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad P=Ad^{3·6}\div L^{1·7}\,.$$

The value of A for rounded ends is 14·9 tons, and for flat ends 44·16 tons. Similarly for hollow pillars of internal diameter d, we have

$$9. \quad . \quad . \quad . \quad . \quad . \quad . \quad P=A(d^{3·6}-d_{,}^{3·6})\div L^{1·7}\,,$$

where for rounded ends A is taken as 13 tons, and for flat ends 44·3 tons.

When the length is less than thirty times the diameter, let P be the ultimate load calculated by equations 8 and 9, and let $P_{,}$ be the load which would crush a short block of the same sectional area S; *i.e.*, let $P_{,}=49S$; let $P_{,,}$ be the actual ultimate strength, then, according to Professor Hodgkinson's experiments,

$$10. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad P_{,,}=\frac{4PP_{,}}{4P+3P_1}\,.$$

For rectangular struts of oak and pine, the smallest side being denoted as before by $d_{,,}$ Hodgkinson gives the formula—

$$11. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad P=A\frac{d^2}{l^2}S\,,$$

where A=3,000,000 lbs. The same unit must be employed for d and l, and S must be expressed in square inches. This formula can only give a rough approximation to the truth. In short beams the formula in § 5 would give a smaller strength than equation 11. This smaller value is, then, the true measure of the strength.

VI. COMPOUND STRUCTURES.

§ 58. Many bridges have been built with superstructures such that the stresses on the several parts or members cannot be computed by the rules hitherto given. These superstructures are generally constructed by superposing two or more types so as to form a compound structure capable of acting at once say as an arch and as a girder. These bridges may be called compound bridges. The designs are usually unworthy of imitation. Mr Robert Stevenson's original design for the Britannia Bridge, in which the great girder would have been partly supported by chains, is an example of this type of structure in which the two parts are clearly visible. Many wooden American bridges are trusses which almost defy analysis, the designs being, however, obviously suggested by an attempt to combine at least two of the three main types of bridges. No advantage whatever is gained by a combination of this kind; on the contrary great disadvantage is almost sure to follow its adoption, namely, that it will be impossible that each part of the structure should, under all circumstances, carry that portion of the load which the designer entrusted to it. For suppose a bridge constructed partly as a girder and partly as a suspension bridge, the girder being very stiff and deep and the chain perfectly flexible with considerable dip. Let the chain and girder be each fit to carry half the passing load. It is perfectly conceivable that the deflections of the two should be so different that the girder would, under the actual load, break before the chain was sensibly strained, or the difference in the relative dip of the chain and depth of the girder might be such as to cause the former to give way first. Even if the two were so designed that at a given temperature each should take the designed share of the load, a change of temperature would entirely alter the proportion borne by the two parts of the structure. A few forms are free from this defect, and these will now be described.

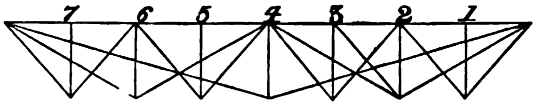


Fig. 85.

§ 59. *Fink Truss.*—This truss (fig. 85) has been much employed in America. The upright struts, numbered 1 to 7, divide the span into eight equal parts. If a weight w rests on the top of each strut the whole truss may be looked upon as made up of seven distinct and independent trusses superposed on one another; strut 4 is used seven times, and is compressed with a total force of $4w$. Struts 2 and 6 are used three times, and each compressed with a total force of $2w$. Struts 1, 3, 5, and 7 are used once, and each compressed with a force w. The stresses on the inclined ties are at once got from the compression of the strut by the resolution of forces; and the stress on the upper member or boom is the sum of all the pulls on the ties resolved horizontally;

the boom is uniformly strained over its whole length. This truss would not be so light as a Warren girder if both were made of the same depth, and if the end struts in the Warren girder did not require to be much stiffened in consequence of their great length. The Fink truss is, however, generally made in practice more cheaply than the Warren girder, because the depth of a girder practically depends on the greatest length of strut which is admissible, and for equal lengths of strut the Fink truss gives a deeper beam than the Warren girder.

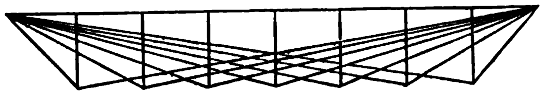


Fig. 86.

§ 60. *Bollman Truss* (fig. 86).—This truss is the result of superposing seven simple frames consisting of a top member, a strut, and two ties. The stresses are easily computed. It is one form of the old false suspension bridge already alluded to (§ 34), with the difference that the top member replaces the horizontal resistance at the points of support. The defect of this truss is that two ties supporting any strut except the central one are of unequal length; expansion or extension, consequently, affects these unequally. It is inferior to the Fink truss.

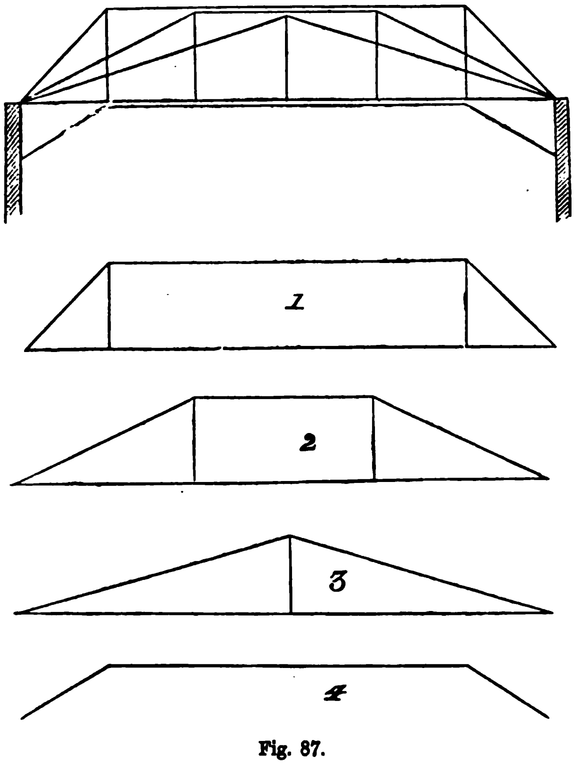


Fig. 87.

§ 61. *Schaffhausen Truss.*—The famous wooden bridge of Schaffhausen (fig. 87, see also § 76) is in its main parts a compound bridge, composed by superposing a series of simple frames of the type shown in 1, 2, 3, and 4. Nos. 1 and 2 are imperfect frames, *i.e.*, if the joints were flexible they would collapse in consequence of the want of the diagonals across the centre parallelogram. The stiffness of the joints supplies this want.

§ 62. *Dredge's Suspension Bridge.*—This bridge differs from the usual suspension bridge in having the suspending rods inclined, and in the use of a lower member, which may be a compression member transmitting a thrust to the piers, as in 88*a*, or a tension member, as in 88*b*, with a maximum tension at the centre. Fig. 88*c* and fig. 88*d* show the reciprocal figures corresponding to those two cases. This bridge is somewhat stiffer than the ordinary suspension bridge, but is far inferior to the complete framed bridge.

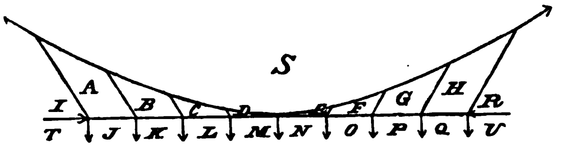


Fig. 88*a*.

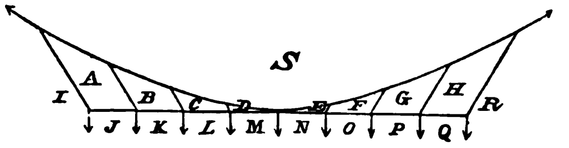


Fig. 88*b*.

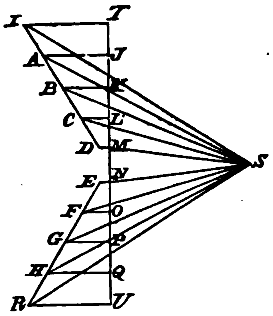


Fig. 88*c*.

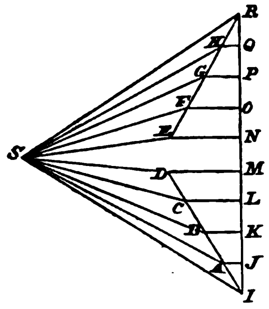


Fig. 88*d*.

§ 63. *Arch or Suspension Bridge, hinged at Abutments and Centre.*—Figs. 89 and 90 show two designs of considerable merit, consisting of two frames (the shape of which might vary considerably) hinged together at A, and supported on hinges at B and C. The direction of the sustaining forces at A and B is to be determined as for a simple pair of beams hinged at A, B, and C. Let *w* be the whole weight carried by the two beams, and let the

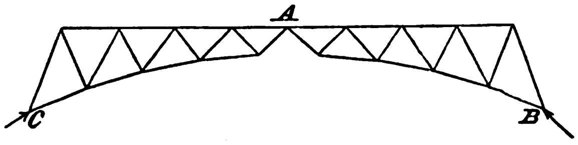


Fig. 89.

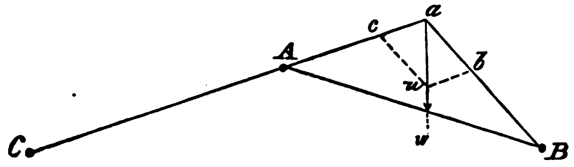


Fig. 89*a*.

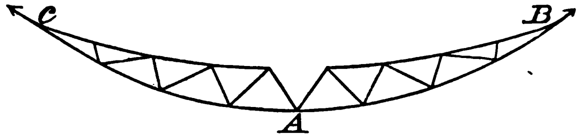


Fig. 90.

centre of gravity of this weight lie in the vertical line *wa*, fig. 89*a*; let the point *a* be the point where the prolongation of the less loaded beam AC cuts this vertical, then the thrust at B and C is obtained by resolving the force *au* equal to the whole weight into the two components *ac* and *ab* in the direction of the lines *a*C and *a*B. This thrust being known, the stress on each member of each frame can be easily computed by a reciprocal figure or otherwise. This form is only inferior to the true framed arch or suspension bridge inasmuch as it is incapable of balancing the thrust due to the passing load on neighbouring spans. It is superior to the framed arch and suspension bridge inasmuch as it cannot be strained by any change of temperature

VII SUBSTRUCTURE.

§ 64. *Preliminary.*—The substructure of a bridge comprises the piers, abutments, and foundations. These portions of the bridge usually consist of masonry in some form, including under that general head stone masonry, brickwork, and concrete. Occasionally metal work or woodwork is used for intermediate piers.

When girders form the superstructure, the resultant pressure on the piers or abutments is vertical, and the dimensions of these are simply regulated by the sufficiency to bear this vertical load.

When arches form the superstructure, the abutment must be so designed as to transmit the resultant thrust to the foundation in a safe direction, and so distributed that no part may be unduly compressed. The intermediate piers should also have considerable stability, so as to counterbalance the thrust arising when one arch is loaded while the other is free from load.

For suspension bridges the abutment forming the anchorage must be so designed as to be thoroughly stable under the greatest pull which the chains can exert. The piers require to be carried above the platform, and their design must be modified according to the type of suspension bridge adopted. When the resultant pressure is not vertical on the piers these must be constructed to meet the inclined pressure. In any stiffened suspension bridge the action of the pier will be analogous to that of a pier between two arches.

§ 65. *Stability.*—When the magnitude and direction of the thrust borne by a pier or abutment at the springing are known, the stability of any series of masonry blocks forming the pier or abutment may be studied by drawing lines showing the direction and magnitude of the resultant force on each joint. This may be done as for the voussoirs of an arch. The thrust on the upper block may be compounded with the weight of that block, the resultant compounded with the weight of the next, and so forth, until the direction and magnitude of the thrust on the rock or earth foundation is determined.

A better method of making the drawing is shown in fig. 91; find the centre of gravity $C_,$ of block 1, the centre of gravity $C_{,,}$ of blocks 1 and 2 treated as a single mass, similarly $C_{,,,}$ for blocks 1, 2, and 3 Let AT be the direction of the thrust on the top block, and $C_,B_,$ a vertical line though $C_,$ cutting AT in $B_,$; let $B_,D_,$ be the direction of the resultant of t, the thrust acting in the line AT, and $w_,$ the weight of the first block acting in the line $C_,B_,$; and let $D_,$ be the point where the direction of this resultant cuts the first joint; similarly let $B_{,,}D_{,,}$ be the direction of the resultant of t and the weight $w_, + w_{,,}$ of the first two blocks; $B_{,,,}D_{,,,}$ the direction of the resultant of t compounded with the weight $w_, + w_{,,} + w_{,,,}$ of the three first blocks, &c., &c. This method of proceeding gives the direction and magnitude of each force and centre of pressure D independently of the values obtained for the preceding joints. For stability the line BD must not make a greater angle with the normal to the joint than the angle of repose; and the point D must nowhere fall beyond the edge of the joint; for strength and safety the point D might be required to fall within the middle two-thirds of the joint, or within the middle three-quarters. The theory by which the joints furthest from the centre of pressure would open when the centre of pressure leaves the middle third cannot apply to such a structure as a masonry abutment, all parts of which are bonded together. Professor G. Fuller calculates the thickness of abutments by the following empirical rule, deduced from many practical examples:—Let c = half the span in feet; d = versed sine in feet; t = thickness of abutment at springing; then, for flat arches, in which the span does not exceed 150 feet, and the ratio of the rise to the span is not less than 116, we may write $t = \cdot 17\left(\frac{c^2}{d} + d\right)$. From the springing to the base the abutment may have a batter of 1 in 4. This gives an abutment the cubic capacity of which will be sufficient, but it may with advantage be divided into abutment proper and counterforts; in semicircular arches t should be taken as the thickness of the abutment at a height above the springing equal to two-thirds of the radius. The maximum intensity of stress on the stone at the edge F might be approximately found by the theory explained in § 8. This theory finds a useful application in calculating the maximum intensity of stress which a given foundation might produce on the earth or rock supporting it. Thus, let the section of the foundation under consideration be 1 foot in breadth, 8 feet in length from F to G, as shown in plan, fig. 92; let the centre of pressure D be 2 feet from FF, and let the total resultant thrust be about 16·5 tons, inclined so that the horizontal component is 4 tons and the vertical component 16 tons, then the mean vertical intensity of pressure per square foot is $\frac{16}{8} = 2$, and the maximum intensity along the line FF, is by equation 2, § 8—

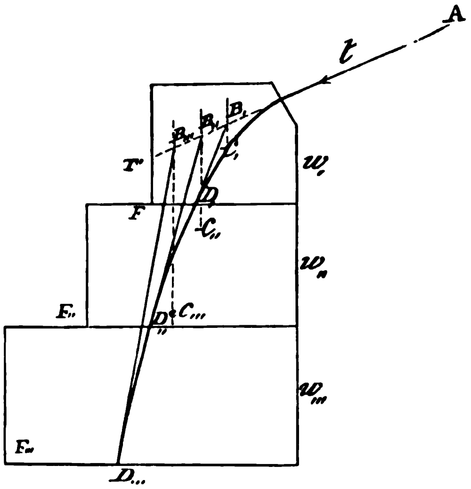


Fig. 91.

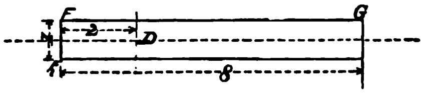


Fig. 92.

$$2\left\{1 + (4 \times 2 \times 8)\frac{12}{512}\right\} = 5,$$

this maximum intensity being 2½ times as great as the mean intensity. Obviously, although for the sake of appearance the courses of masonry in the abutment of an arch may be left horizontal in the face, the stability is increased by inclining them at a proper angle, so that they lie normal to the thrust.

The stability of abutments may be tested by taking moments round points in the joints selected as the points beyond which the thrust must not come. The same methods apply to the anchorages of suspension bridges, and to intermediate piers, which are intended to take a given horizontal thrust. When metal or wooden piers are adopted their weight will generally be insignificant, and such as may be neglected in calculating their stability. Metal-work piers or wooden piers usually consist of wrought or cast-iron frame work, and the stress on each part of the frame, as well as the resultant stress on the foundations where each upright member reaches it, is easily calculated by the method of reciprocal figures or otherwise.

Occasionally metal piers are continuous metal structures, such as cast-iron cylinders. The maximum intensity of stress can then be calculated by resolving the thrust on the upper part of the pier into a horizontal and a vertical component, calculating the bending moment produced on each horizontal cross section by the horizontal component, and adding the intensity of stress caused by this bending moment to the mean intensity caused by direct compression. The manner in which any metal-work pier is held by its foundation against a bending moment will require special consideration; the resultant pressure should always fall well within the base.

§ 66. *Practice.*—In the design of the usual masonry bridge the thickness of the pier is generally determined by practical considerations. In small arches the pier is made thick enough to allow the two rings of the two abutting arches to spring from the pier without interfering with one another, a clearance of about half a brick being often allowed between the two rings. In larger arches the piers will generally be found to vary in thickness from $\frac{1}{8}$th to $\frac{1}{10}$th of the span, with a slight batter (*i.e.*, with walls spreading outward towards the base). In very old bridges piers are sometimes found equal in width to the opening of the arch. In large bridges, or with very high piers, care must be taken that the pressure per square foot on the masonry or foundation does not exceed a safe value. The brickwork in the piers of Charing Cross bridge is subject to a compression of 9 tons per square foot; four or five tons is a much more usual load. Eight tons per square foot may be considered a maximum for rubble stone-work, and perhaps 20 tons for the best dressed ashlar. Strong concrete may be trusted with 3 tons; firm rock foundations with 9 tons, soft sandstone with 2 tons, and firm earth with from 1 to $1\frac{1}{2}$ tons. The depth of the first course below the surface (on dry land) should not be less than 3 feet in sand and 4 feet in clay.

When framework, either of wood or iron, is used as a pier, care must be taken by cross-bracing to provide against the effect of wind and vibration.

§ 67. *Site.*—The site proposed for a pier must be carefully examined by borings; the ground should be uniform, for if a pier rests partly on one formation and partly on another, unequal settlement will certainly occur, even if the weaker formation be such as would have been amply strong enough to bear the pressure had the pier been wholly founded upon it. Solid rock may be considered the best foundation, but where rock is broken up by cracks or other inequalities it is inferior to such formations as uniform gravel, chalk, and some kinds of sand and clay. These foundations may be described as incompressible. The worst foundations are afforded by those formations which can be compressed or squeezed out sideways by the imposition of weight. Muddy earth, certain clays, and certain sands are of this nature. Alternate beds of stone and slippery clay are very treacherous. The foundation should be dressed level so that the masonry may everywhere start from the same height, and therefore settle equally. Unavoidable inequalities are better filled up by concrete than by masonry.

For foundations in water it is very important that the ground should not be such as can be scoured away by the current or wash of the water; many bridges have failed by the undermining of the piers due to this cause. Special precautions, to be presently described, must be taken against the effects of the scour if the soil itself is not of a sufficiently resisting nature. The piers must be so placed and formed that the obstruction to the flow of water may be as small as possible and the effect which the piers will have in altering the level of the stream above and below the bridge must be considered. Data as to the maximum flood waters to be provided for must be examined; and provision must in some climates be made against ice by suitable cut-waters or fenders, an example of which is given in fig. 93, showing a pier of the Victoria Bridge, Montreal.

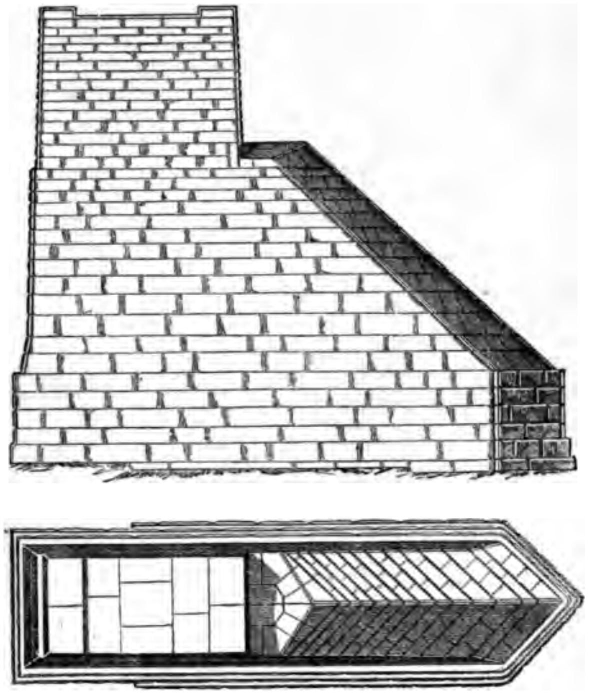


FIG. 93.—Side Elevation and Plan of Pier.

§ 68. *Mode of Founding in Water.*—The chief difficulty met with by an engineer about to erect a large bridge over a deep stream is to secure a sound foundation for the piers. The following are some of the principal methods of building piers in or under water:—

Cofferdams.—Cofferdams are embankments or dams which surround the site so as to exclude the water from it. They are formed in general by driving two rows of piles round the site so as to enclose between them a water-tight wall of clay puddle; in depths of less than 3 or 4 feet, where there is little current, a simple clay dam may be used. In greater depths, the timber walls consist of guide piles at intervals, with some form of sheet piling between them; in extreme depths the timber walls may be composed of stout piles driven in side by side all round. The dam must be sufficiently strong to bear the pressure of the water against the outside when the space enclosed has been pumped dry. Rankine states that the common rule for the thickness of a cofferdam is to make it equal to the height above ground if the height does not exceed 10 feet, and for greater heights, to add to 10 feet one-third of the excess of the height above 10 feet. The "Cours de Ponts" at the school of the Ponts et Chaussées, states that a cofferdam need never be made of greater thickness than from 4 to 6 feet, as the interior can always be sufficiently stayed inside. This method of founding is now seldom practised; it is costly and causes great obstruction in the stream.

Caissons.—Some foundations have been constructed as follows:—A level or nearly level bed was prepared in the stream by digging or by driving piles and sawing off the heads at a uniform depth; a huge timber box, called a caisson, was then filled with masonry, and sunk on the foundation thus roughly prepared. This method is now abandoned. It was peculiarly liable to danger from the scour of the stream. The name caisson is also sometimes applied to a mere frame with wooden walls which is floated to the site of the pier, and there sunk so as to form an enclosure, inside which concrete can be shot and can set undisturbed by the wash of the water.

Concrete in a shell is a name which might be applied to all the methods of founding a pier which depend on the very valuable property which strong hydraulic concrete possesses of setting into a solid mass under water. The required space is enclosed by a wooden or iron shell; the soil inside the shell is removed by dredging, or some form of mechanical excavator, until the formation is reached which is to support the pier; the concrete is then shot into the enclosed space from a height of about 10 feet, and rammed down in layers about 1 foot thick; it soon consolidates into a permanent artificial stone. The shell, unless of small size or very strong form, requires to be braced to meet the outward pressure of the concrete. The concrete used for this purpose is often called béton, to distinguish it from inferior mixtures used for foundations on land. It may consist of angular stones $1\frac{1}{2}$ to 2 inches diameter, mixed with strong hydraulic mortar in the proportion of from one to two volumes of stones with one of mortar; the final volume may be from $\frac{5}{9}$ to $\frac{5}{8}$ of the unmixed materials. Béton used at Biarritz consisted of one part Portland cement, two parts sand, and three parts broken stones; at Genoa one part rich lime, two parts pozzuolana, three parts broken stones.

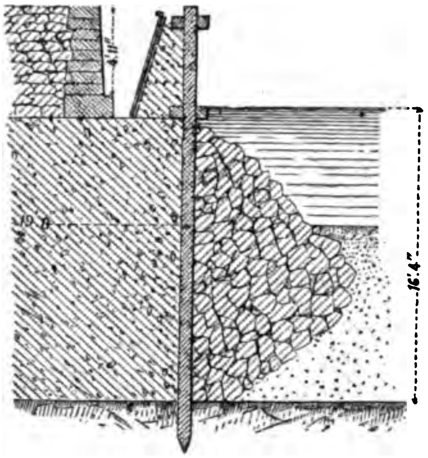


Fig. 94.

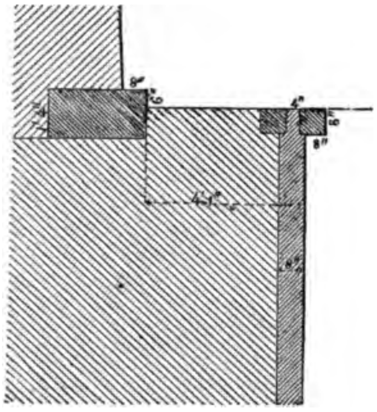


Fig. 94a.

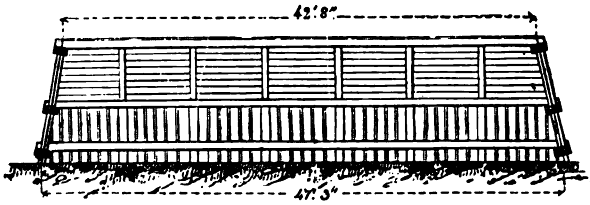


FIG. 95.—Caisson used for a Bridge over the Cher.

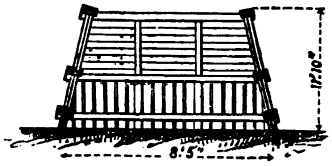


Fig. 95a.

Fig. 94 shows a section of foundations constructed by filling a casing of piles with concrete in this manner; the shell is protected against scour by large stones heaped round the outside, part of the loose earth having been removed by digging. Fig. 94a shows the manner in which the concrete foundation was finished after removal of the temporary external wall shown in fig 94.

Figs. 95 and 95a show a longitudinal elevation and cross section of a wooden shell, or caisson without a bottom, intended to be sunk to receive concrete. Cast-iron guide piles and sheet piling are also used for this class of foundation.

Cast-iron tubular shells are now frequently employed; the tube forms a large hollow pile, which may descend into the ground by its own weight, or by added weight while the soil inside is removed by some kind of dredge or excavator, such as Milroy's, worked from the surface. When the lower edge of the shell has penetrated the formation to be used as a foundation, the water inside may be pumped out if the soil forms a water-tight joint, or the shell may simply be filled up with concrete shot into the enclosed water. The piers of Charing Cross bridge (fig. 96) were constructed in this way; the excavation inside the tube (14 feet diameter), was carried on by divers with helmets until the shell had entered a few feet into the London clay. The water was then pumped out and excavation continued; the cylinders were loaded with about 150 tons to sink them to the final depth.

Compressed air is now very generally employed inside a metal shell for those foundations in which the excavation requires the presence of workmen at the bottom of the shell. The metal shell is open at the bottom, but air-tight and water-tight at all other points; there is a chamber called an *air-lock* at the upper part. This "air-lock" serves for the exit and entrance of the workmen and materials; the air in this comparatively small space is lowered to the pressure of the atmosphere before the chamber is opened for the passage of men or materials to the open air; the air is again compressed in the air-lock before it is opened for communication with the body of the shell in which the air is permanently kept at such a pressure as will keep the water down to the required level. The shell thus acts as a diving bell acts. It is found that men cannot in general be safely employed under a greater pressure than two atmospheres above the ordinary atmospheric pressure, corresponding to a depth in water of about 65 feet. The centre pier of Saltash Bridge was, however, in 1855 by this plan carried down to a depth of 87 feet 6 inches below high water. Recently the foundations of the St Louis bridge at Cincinnati have by the same method been established at a depth of

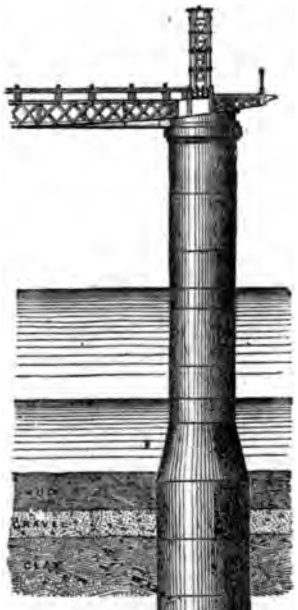


FIG. 96.—Cylinder, Charing Cross Bridge.

110 feet from the surface of the water. Figs. 97 and 97 A show the pier of the bridge of Argenteuil. Excavation was carried on in the lower chamber, the roof of which was very strongly built of metal, and served as the foundation for masonry and concrete built up round an inner tube, serving as a passage for men and materials to the upper chamber and "air lock." This lock is formed of two concentric cylinders of 3·9 feet and 10·8 feet diameter respectively (fig. 97*a*). The annular space was divided by two vertical partitions; the doors of communication (which were air-tight) were 1·8 feet wide and 2·14 feet high. A small engine worked the hoist by a stuffing-box passing through the shell. A safety-valve is of course required. The upward pressure of the air requires to be counterbalanced by weights. In the Argenteuil bridge this necessary weight was afforded by the masonry built in the tube as the tube sank. This plan seems preferable to the method of loading the shell externally with pig or railway iron. Frequently, owing to the tenacious nature of the soil, the water cannot be driven out below the tube, and in that case a syphon must be provided passing out at the top. The Argenteuil tube was sunk at a mean rate of about 18 inches per diem. This method is not confined to cylindrical tubes. Fig. 98 shows the method employed in building the piers of the bridge at Kehl over the Rhine. In this case four rectangular working chambers were sunk side by side and bolted together; each chamber communicated with the surface by two air-passages, and one central elliptical passage which remained full of water. This central passage served for the exit of the excavated material. A mass of concrete was built resting on the working chambers, and contained by wooden framework. The concrete was added at the top above water as the foundations gradually sank. At Mantes and Chalons wrought iron caissons, shaped like the usual masonry piers, have been sunk by analogous methods.

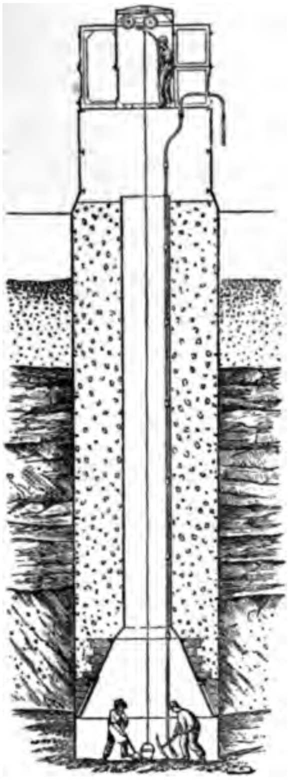


FIG. 97.—Foundations, Bridge of Argenteuil.

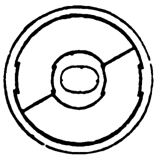


Fig 97*a*.

The method of sinking cylinders by compressed air was invented by Mr Triger in 1841, and was first used on a large scale by Mr Hughes at Rochester. The tubes at this bridge were designed to be sunk by having the air *exhausted* inside the tube, a system invented by Dr Potts.

§ 69. *Piles* are used either to enclose a space or to bear part of the weight of a structure; for the former purpose a wooden pile may be a round or square pointed piece of timber, 6 or 9 inches in diameter and 8 or 12 feet long Bearing piles may be of any dimensions which can practically be procured, and several lengths of timber are often jointed so as to form one long strut. The point is armed with metal, and the head protected by a metal ring, which prevents it from spreading when struck by the rammer which drives the pile. Bearing piles are usually placed at a distance from centre to centre not less than 2 feet 6 inches or more than 4 feet; 3 feet is a common distance. The diameter of bearing piles varies from 9 inches to 20 inches; a pile may be considered to be driven home when, with thirty blows of a ram weighing 800 ℔ and falling 5 feet, it does not move $\frac{1}{5}$th of an inch (Rankine). A French rule gives as a limit $\frac{1}{6}$ inch motion with twenty-five blows from a ram weighing 6 cwt. and falling 4 feet 3 inches. A pile which does not move more than this will bear from 600 to 1000 ℔ per square inch. This would give a load of 50 tons for a 13-inch pile; if, as is more usual, the load be only 8 or 10 tons for a 13-inch pile, the ultimate rate of descent may be three, four, or five times as much as the above.

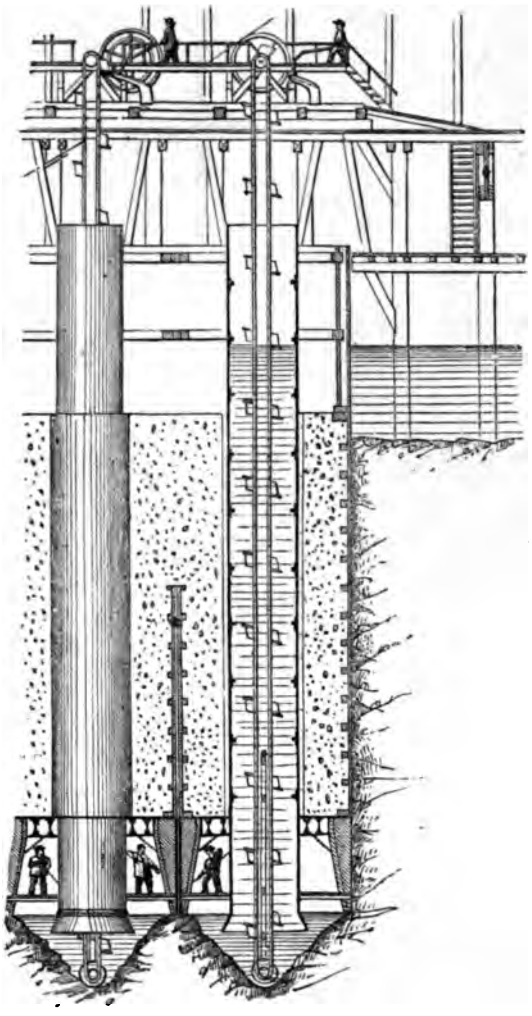


FIG. 98.—Foundations, Bridge of Kehl on the Rhine.

Piles are used as foundations in those grounds which are compressible, or which would be squeezed out from beneath masonry under the weight to be borne. The wooden bearing piles are usually sawn off so that all the heads may be level, and a wooden grating or platform rests on the heads, over which the concrete or masonry pier may be built; in other cases the piles come up for some distance into the concrete. An external row of wooden piles is not unfrequently employed as a precaution against scour, but these should always be further protected by a stone bank, which will continue to protect the pier if the piles decay. A more thorough protection against scour is provided by covering the centre bed of the river with a concrete or stone

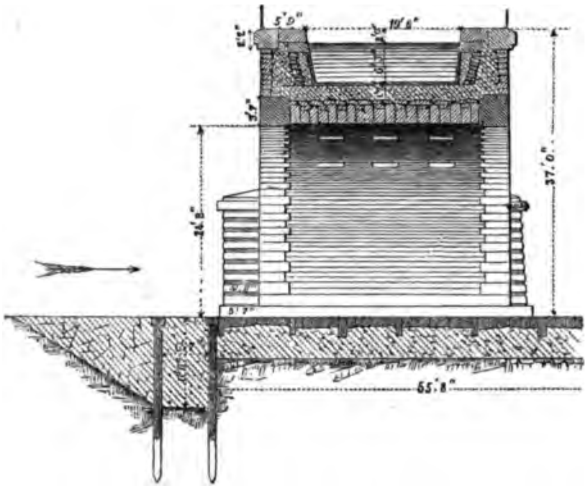


FIG. 99.—Aqueduct of Guétin over the Allier.

floor, as in fig. 99, representing a pier of the aqueduct crossing the Allier at Guétin.

Cast-iron guide piles (usually hollow tubes) are used for the same purpose as wooden guide piles. They may be cast with grooves down one edge in which sheet piling is held. Cast-iron and wooden piles are frequently used as part of the open-work metal or wooden framing of piers. In situations where these piles are not liable to injury by ice, floating timber, or barges, this construction is very economical. Fig. 4, Plate XIX. shows the Crumlin viaduct, with piers of this character, the construction of which will be more fully described in paragraph 79.

Fig. 100 shows the Portage bridge (234 feet high) spanning the Genesee River, in the State of New York. The piers are large wooden frames.

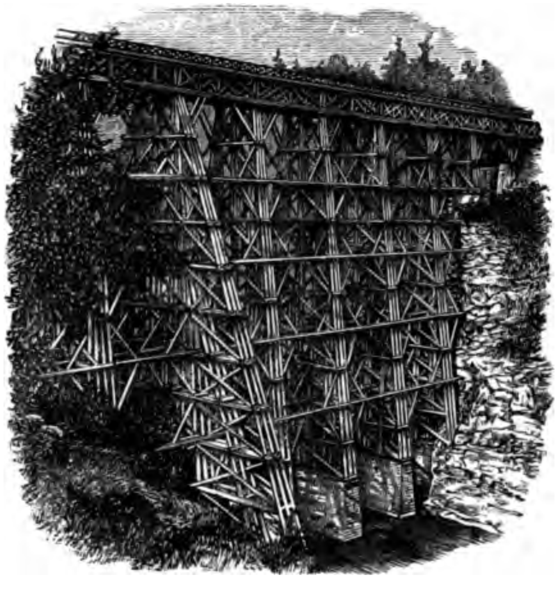


FIG. 100.—Portage Bridge.

§ 70. *Screw piles* are cast-iron piles which are screwed into the soil instead of being driven in. At their end is fixed a blade of cast-iron from two to eight times the diameter of the shaft of the pile; the pitch of the screw varies from one-half to one-fourth of the external diameter of the blade. The pile is turned by levers radiating from its head. In one example of their use (Rankine) the pile was screwed in by four levers, each 40 feet in length, with eight bullocks harnessed to it. The screws were 4 feet 6 inches in diameter, and the working load borne by them was 100 ℔ per square inch. The piles were screwed from 20 to 45 feet deep in earth.

Disc piles have been used in sand. These piles had a flat flange at the bottom, and water was pumped in at the top of the pile, which was weighted to prevent it from rising. Sand was thus blown or pumped from below the piles, which were thus easily lowered in ground which had baffled all attempts to drive in piles by blows. In ground which is of the nature of quicksand, piles will often slowly rise to their original position after each blow.

§ 71. *Wells.*—In some soils foundations may be obtained by the device of building a masonry casing like that of a well and excavating the soil inside; the casing gradually sinks and the masonry is continued at the surface. This method is applicable in running sands. The interior of the well is generally filled up with concrete or brick when the required depth has been reached, but in some cases a mere floor or inverted arch would be preferable.

VIII. EXAMPLES.

§ 72. The task of selecting a limited number of bridges which shall represent the gradual progress in the art of construction and illustrate our present practice is one of much difficulty. Many very admirable and interesting structures must necessarily be passed over in silence, and space will not admit of full details being given even of those bridges which are noticed.

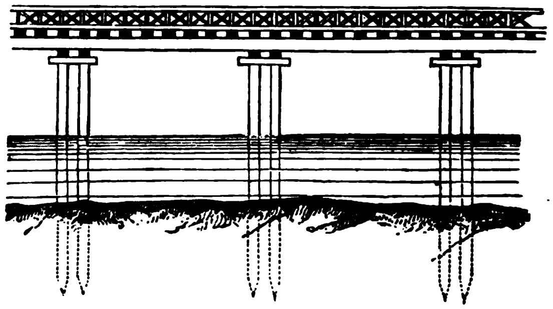


FIG. 101.—Pons Sublicius.

§ 73. *Bridges built before the year* 1000 A.D.—Herodotus mentions a bridge erected by Nitocris over the Euphrates at Babylon. It appears to have consisted of stone piers connected by planking, which was removed at night. The river was diverted to allow the piers to be built. Diodorus Siculus ascribes the work to Semiramis.

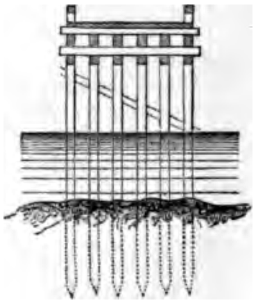
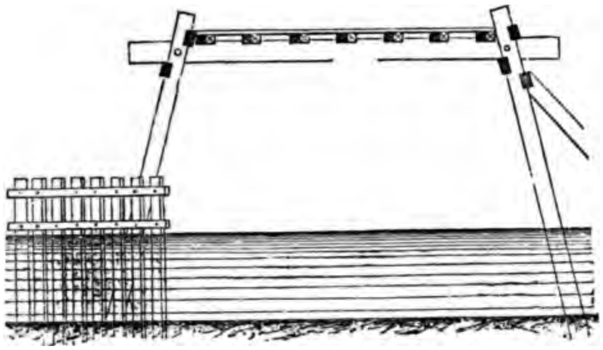


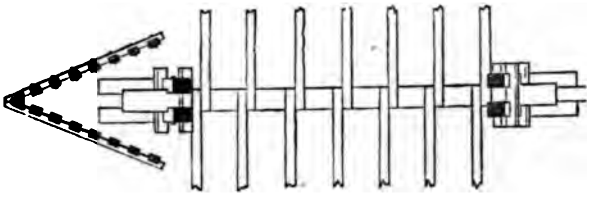
Fig. 101a.

Sublician Bridge.

The first bridge constructed at Rome was called the *Pons Sublicius*, or wooden bridge (*sublica* meaning a stake or pile). It is said to have been built by Ancus Martius, and rebuilt by the chief priests, who from this circumstance were called "Pontifices." Fig. 101 shows the design of this bridge as restored by Colonel Emy (*Traité de l'art* *de la charpenterie*) from the descriptions given in the historians. This was the bridge defended by Horatius Cocles.



Cross Section at Pier.



Plan at Pier.

FIG. 102.—Bridge thrown across the Rhine by Julius Cæsar.

Fig. 102, also taken from the work of Colonel Emy, is intended to represent the design of the bridge thrown across the Rhine in ten days by Julius Cæsar (*De Bell. Gall.* iv. 17).

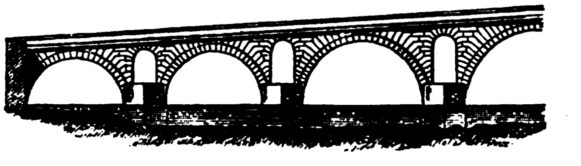
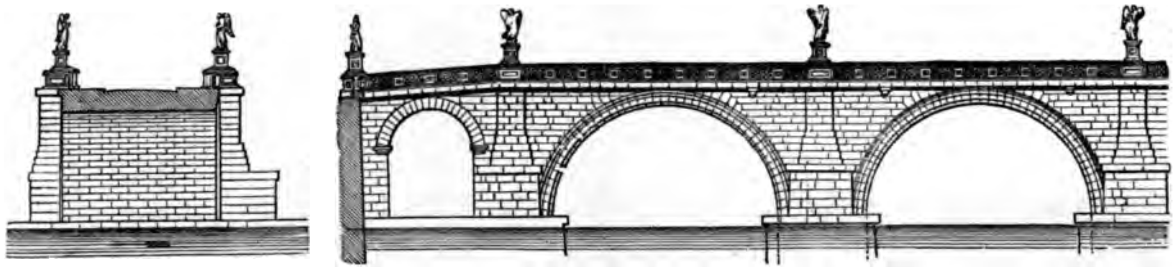


FIG. 103.—Pons Milvius.

Pons Milvius.

The *Pons Milvius* (fig. 103), now Ponte Molle, was built a mile and a half from Rome by the Censor Ælius Scaurus, about 100 B.C. Some part of the first bridge is supposed to remain, although it has been altered from time to time. The arches vary in their opening from 51 feet to 79 feet 9 inches; the waterway between the piers is 413 feet 3 inches; the breadth of the bridge, 28 feet 9 inches; these dimensions are given on the authority of Cresy (*Encyclopædia of Civil Engineering*). The following bridges also crossed the Tiber at Rome:—The *Pons Palatinus*, which stood on the site of the present Ponte Rotto; the *Pons Fabricius* and *Pons Cestius*, which still remain; the *Pons Janiculanus*, which occupied the site of the modern Ponte Sisto; the *Pons Vaticanus*, which has disappeared; and the *Pons Ælius*, built by Hadrian (13 A.D.), now the bridge of St Angelo. This bridge (fig. 104) was repaired by Popes Nicholas III. and Clement IX. The largest arch has a span of 62 feet 4 inches, and the width of the bridge is 50 feet 9 inches.

Other Roman bridges.



Cross Section. Elevation.

FIG. 104.—Bridge of S. Angelo.

Trajan's Bridge.

The bridge erected by Trajan (104 A.D.) across the Danube, just below the rapids of the Iron Gate, has been the subject of much controversy. The drawing (fig. 105) was originally taken from a bas-relief on the Trajan column at Rome. A description of the bridge is given by the ancient historian Dion Cassius, who states that the bridge had twenty piers of hewn stone, 150 feet high and 60 feet wide, with openings between them of 170 feet, spanned by arches. Doubt has been thrown on the accuracy of this description, because the design shown in fig. 105 is obviously unsuited to a span of 170 feet; nevertheless thirteen piers are still visible out of the twenty, according to Murray's *Handbook* The writer has not been able to find any accurate measurement of the width between these piers, but as the *Handbook* speaks of the length of the bridge as perhaps 3900 feet, and as the Conte Marsigli, writing from personal observation, in a letter to Montfaucon, gives the total length as probably 3010 feet, there can be no doubt that the spans were very considerable, and that the representation of the design in the bas-relief is almost wholly conventional. The one point as to which it gives clear information, not supplied elsewhere, is that the superstructure was of wood. The piers seem to have been founded by sinking caissons. Murray's *Handbook* gives the depth of the river as 18 feet. Apollodorus of Damascus was the architect of this remarkable bridge. The bridge at Rimini, built during the reign of Augustus, was especially admired by Palladio (Rondelet, *L'Art de bâtir*). The bridge at Narni, on the road from Loretto to Rome, also built by Augustus (Montfaucon), and the bridge of Alcantara over the Tagus, built in the reign of the Emperor Trajan, are often cited as remarkable works.

Rimini. **Narni.** **Alcantara.**

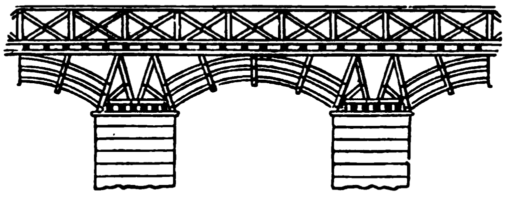


FIG. 105.—Trajan's Bridge

St Chamas. **Ponte Salaro.**

The Romans frequently adorned their bridges with a triumphal arch. A small example of this kind of bridge at St Chamas, in France, is shown in fig. 106 (Cresy's *Encyclopædia of Civil Engineering*). The span of the arch is 42 feet, and the voussoirs are 3 feet 5 inches deep. Fig. 107 hows the bridge of Narses, built in the 6th century, and which carried the Via Salaria across the Anio or

Teverone. This bridge was blown up during the panic caused by the approach of Garibaldi to Rome in 1867.

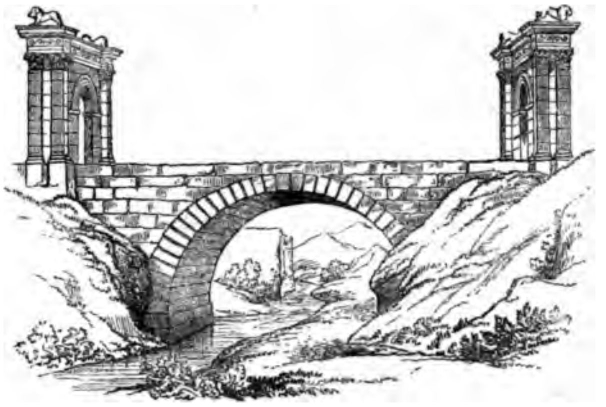


FIG. 106.—Bridge at St Chamas.

We see here, perhaps, one of the earliest examples of the castle built to protect the bridge against an enemy or to enforce payment of a toll,—the bridge and castle of mediæval romance.

§ 74. 1000 to 1300 A.D.—A very bold arch over the Serchio near Lucca is shown in plate 58 of Hann and Hosking's treatise, with the approximate date of 1000 A.D., but the authority for this date is not given. The span of the arch is 120 feet, and the roadway, which stands at a height of more than 60 feet above the water level, is only 9 feet wide; in fact the arch is little more than a broad wall. Owing chiefly to the excellence of the mortar employed, this arch withstood a flood which rose nearly 30 feet above the springing of the arch. This structure is one of the many "Devil's Bridges."

In the year 1178 a famous bridge was begun over the Rhone at Avignon by Saint Benezet, the head of one of certain religious confraternities, which undertook the building or repair of bridges during the Middle Ages, and were called Fratres Pontis or Hospitalarii Pontificea. The bridge was finished in 1188. Four arches still remain, and are remarkable in having an elliptical outline with the radius of curvature smaller at the crown than at the haunch, a form which accords more truly with the linear equilibrated arch than the modern flat ellipse with the largest radius at the crown. A description and drawing of this remarkable bridge will be found in the *Dictionnaire raisonnée d'Architecture* of M. Viollet-le-Duc.

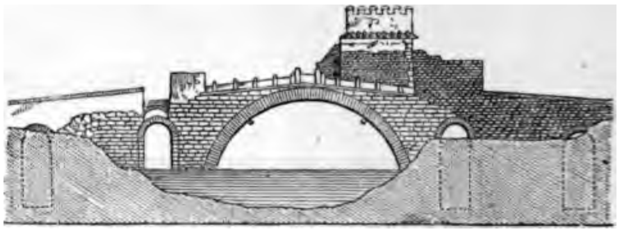


FIG. 107.—Ponte Salaro.

A religious confraternity, founded in the first instance by a certain Mary, the maiden daughter of a ferryman, is said to have built a timber bridge near the site of the present London Bridge, but it seems quite uncertain when the first bridge over the Thames was built. There is little doubt, however, that it was of timber, and had frequently to be reconstructed. Stow, in his *Survey of the Cities of London and Westminster*, gives a description of the building of the first stone structure, commonly called Old London Bridge, Plate XVIII. fig. 1.[1]

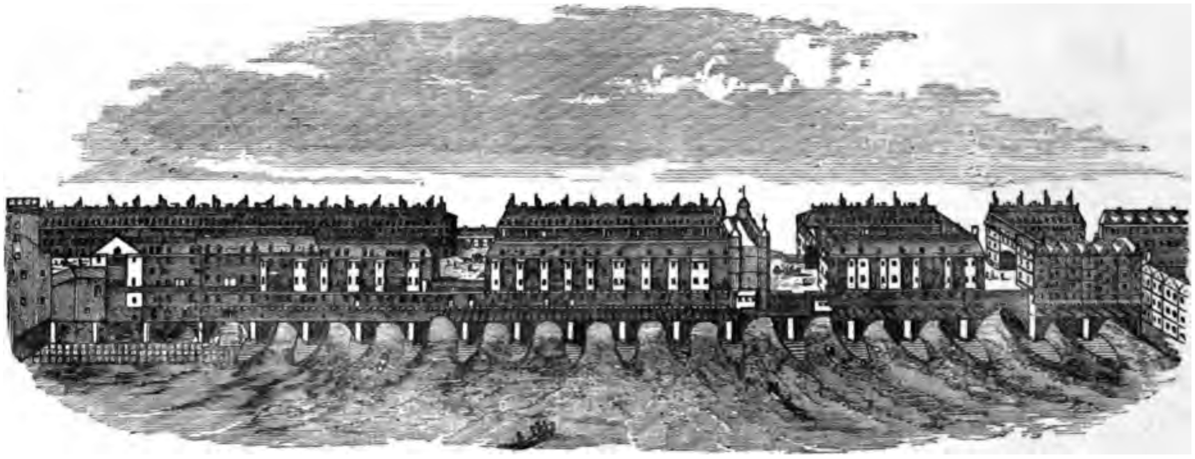


Fig. 108.—Old London Bridge, A.D. 1700.

Heavy repairs were frequently necessary, and the timber houses built on the bridge were often burnt down; yet the main structure appears to have remained unaltered until the beginning of this century. It does not seem improbable that Peter of Colechurch and Saint Benezet may have been in communication with one another, both being heads of religious bodies engaged in similar works at the same time. Their letters to one another would interest engineers. A French brother Isembert, from Saintes, succeeded Peter as engineer for London Bridge.

Stow describes the partial rebuilding of the timber houses in 1645:—

"The building was of timber, very substantial and beautiful, for the houses were three stories high, besides the cellars, which were within and between the peers, and over the houses were stately platforms leaded with rails and ballasters about them, very commodious and pleasant for walking and enjoying so fine a prospect up and down the river, and some had pretty little gardens with arbours."

---

[1] "About the year 1176 the stone bridge over the river Thames at London was begun to be founded by the foresaid Peter of Colechurch (called priest and chaplain before), "near unto the bridge of timber, but more towards the west, for I read that Botolph's wharf was in the Conqueror's time at the head of London Bridge. The king assisted this work, a cardinal then being legate here; and Richard, archbishop of Canterbury, gave one thousand marks towards the foundation. The course of the river for the time was turned another way about by a trench cast for that purpose beginning, as is supposed, east about Rotherhithe, and ending in the west about Patricksey, now termed Battersea. This work, to wit the arches, chapel, and stone bridge over the Thames at London, having been thirty-three years in building, was, in the year 1209, finished by the worthy merchants of London—Serle Mercer, William Almaine, and Benedick Botewrite, principal masters of work; for Peter of Colechurch deceased four years before the work was finished, and was buried in the chapel builded on the same bridge in the year 1205."

The passage between the houses was made 20 feet; previously it had been but 14 feet, and in some parts 12 feet. These beautiful houses were burned in 1666, when they were replaced by a still finer pile of buildings, with a uniform roadway of 20 feet in width. Fig. 108 shows the bridge as it appeared in 1700.

The piers varied in thickness from 25 to 34 feet, and were raised on strong elm piles, covered with thick planks bolted together. The openings in the arches varied from 10 feet to 32 feet 9 inches. The whole waterway was 336 feet 9 inches, two-thirds of the stream being occupied by piers.

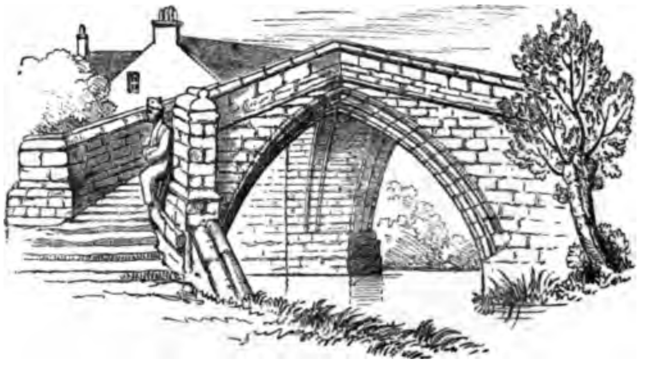


FIG. 109.—Croyland Bridge.

§ 75. 1300 to 1700 A.D.—The strange triangular bridge Croyland. at Croyland (fig. 109) is another example of a bridge probably built for or by a religious body. This structure stands at the confluence of the Welland, the Nyne, and the Catwater drain; three pointed arches, having their abutments at the angles of an equilateral triangle, meet in the middle, giving three watercourses and three roadways. Each arch has three stone ribs, and the nine meet in the centre. Croyland "triangular bridge" is alluded to in a charter of the year 943; from the character of the masonry the present structure is supposed to have been built in the Boston. beginning of the 14th century. A bridge over the Trent at Boston, 1534 feet in length, and consisting of 34 arches, was also built by a religious community under Abbot Bernard.

Saintes. Fig. 110 shows the old bridge at Saintes as M. Viollet-le-Duc considers that it appeared towards the end of the 14th century. The following description is abridged from his *Dictionnaire raisonnée d'Architecture*:—

> "The first gate appeared on the right shore of the river, on the side of the Faubourg des Dames; next came the Roman arch, the upper part of which was crenelated during the Middle Ages; next on the side of the town stood a tower of oval plan, through which the road lay; the town gates with flanking towers closed the end of the bridge. From the first gate to the Roman arch the bridge was of wood, as was also the case between the great tower and the town gates, so that by the removal of this part of the roadway all communication could be cut off between the town and the tower as well as between the bridge and the Faubourg; moreover, the parapets were crenelated, so that the garrison of the town could at will stop all navigation."

Clearly it was quite as important in those days to be able to arrest as to facilitate communication between the two sides of the river.

The architects of the Renaissance showed great boldness Trezzo. and originality in their designs. The largest arch known to have been built spanned the Adda at Trezzo, constructed by order of Bernabò Visconti, duke of Milan (latter half of 14th century. This bridge is described in Hann and Hosking's *Bridges*, as having consisted of "a single arch of granite, very well constructed of stones in two courses, the innermost 3½ feet thick in the direction of the radius, the outermost 9 inches, the span at low water 251 feet; the river rises sometimes 13 feet." The radius of the arch was 133 feet. This noble bridge was destroyed by Carmagnola.

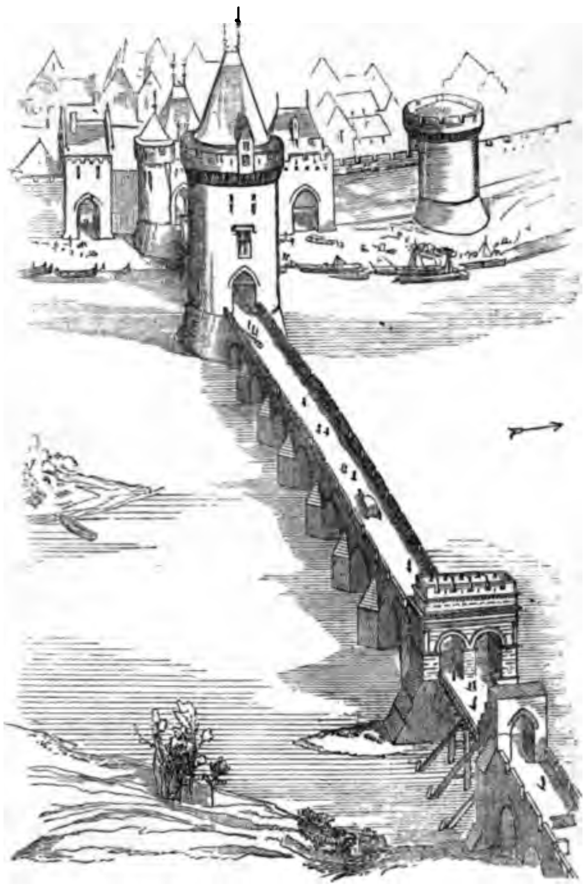


FIG. 110.—Saintes Bridge.

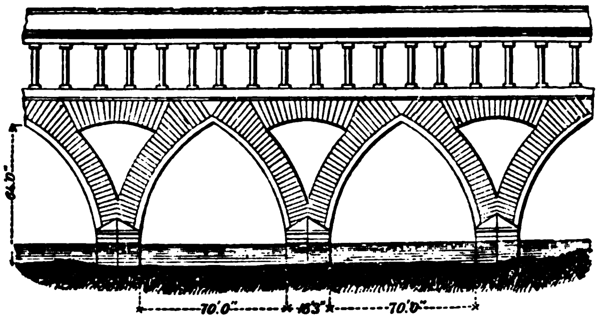


FIG. 111.—Bridge over the Ticino at Pavia.

The covered bridge over the Ticino at Pavia (fig. 111) Pavia. was erected, under Gian Galeazzo Visconti, about the end of the 14th century. This bridge, which still exists, has seven pointed brick arches, each 70 feet in span and 64 feet in height; the depth of the arch ring at the crown is 5 feet 6 inches. The tympanum is pierced; the bricks used in the arches are formed to suit their position, and are hollow in the middle to diminish the weight. The roof of the roadway is carried by 100 rough granite columns.

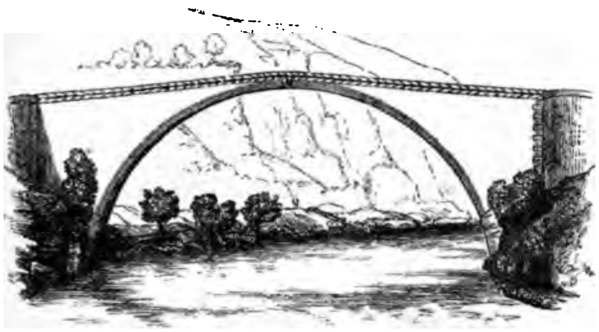


FIG. 112.—Bridge of Brioude.

Brioude.

Fig. 112 taken from Montfaucon's *Antiquité expliquée*, shows the old bridge of Brioude across the Allier in France. Montfaucon and Séguin speak of this as a Roman work, but Gauthey gives the date 1454 for its construction, and names Grenier and Estone as builders without giving his authority. The design of the bridge appears to favour the date given by Gauthey. The span was 183·73 English feet (Rennie, *Proc. I.C.E.*), the arch was a segment of a circle and the height 60 feet, while the width of the bridge was only 16 feet. This bridge fell in 1822.

Rialto, Venice.

The bridge of the Rialto at Venice (fig. 113) was begun in 1588, Antonio da Ponte being the architect. The span of the arch is 91 feet, the height above the water level 24 feet 6 inches, and the width of the footway 72 feet. Erroneous statements are often met with that this bridge was built from a design by Michel Angelo; the mistake has arisen from the misinterpretation of a passage in the works of Vasari. Rondelet, in his *Essai Historique sur le Pont du Rialto*, gives a full account of the rival designs submitted to the senate by Antonio da Ponte and Palladio.

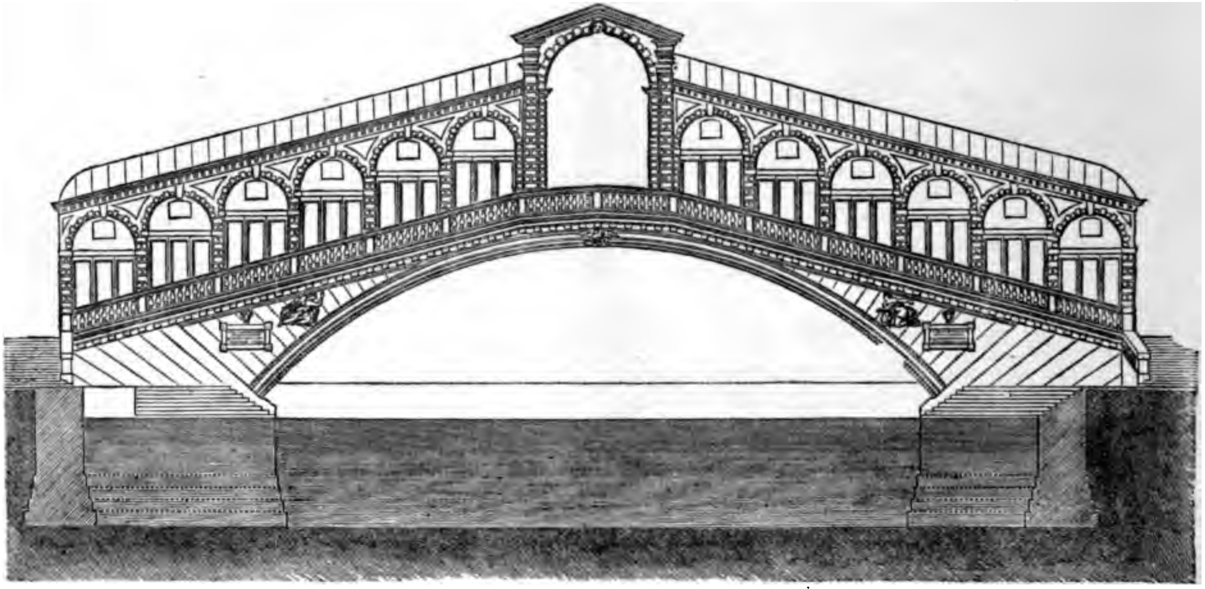


FIG. 113.—Bridge of the Rialto, Venice.

Fig. 114 shows the singularly beautiful "Ponte della Trinità," erected at Florence (1566) from the designs of Ammanati. Those who are curious in such matters may observe with interest the amended design for this bridge, given in Hosking's *Architectural Treatise on Bridge Building*, p. 241, which serves to show how easily a noble design may be spoilt by an alteration in the proportions of its parts.

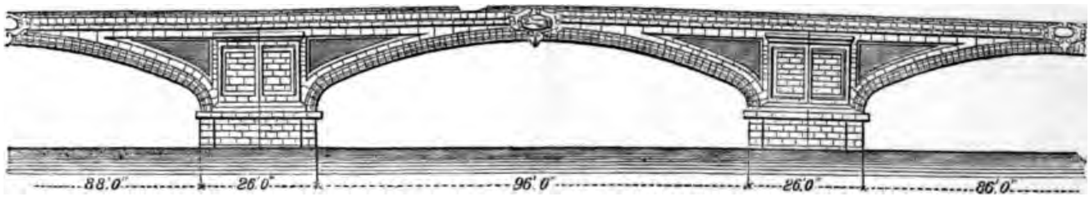


FIG. 114.—Ponte della Trinità, Florence.

A fine bridge over the Ouse at York, erected in the reign of Queen Elizabeth, was taken down some years ago; it is shown in fig. 115. The span of the largest arch was 81 feet (Allen), and the rise 26 feet 3 inches.

The well-known Pont Neuf at Paris was built in 1604. The design has no feature calling for special remark. Fig. 116 shows the bridge over the Conway at Llanrwst in Wales, the design for which was furnished by Inigo Jones in 1634. The middle arch has a span of 58 feet. The structure is easily set in vibration, and is known as the "shaking bridge." Further particulars concerning this and many other old English bridges will be found in Smiles's *Lives of the Engineers*, chap. iii. vol. ii., edition 1874.

Ispahan.

A bridge over the Senderud at Ispahan is described as follows in Heck's *Iconographic Encyclopædia*, with illustrations:—"It (the bridge) is 2250 feet long, 120 feet high, and 156 feet broad; the middle way is 60 feet broad, and the sideways are paved with marble, and the latter lead through arcades, to which the ascent is by stairs in the four towers of the bridge. The bridge has 29 arches of 50 feet span, and the pillars are 25 feet thick.' In the illustration the arches are Moorish, and the covered sideways lofty, with 3 arches of small span over each main arch of the bridge. The design is remarkably fine. Heck calls it the bridge of Barbaruh, and states that it is named from its builder, but it is of unknown antiquity. All the bridges of Ispahan are said, in the 7th edition of the present work, to have been built under Shah Abbas I. (1585 to 1628).

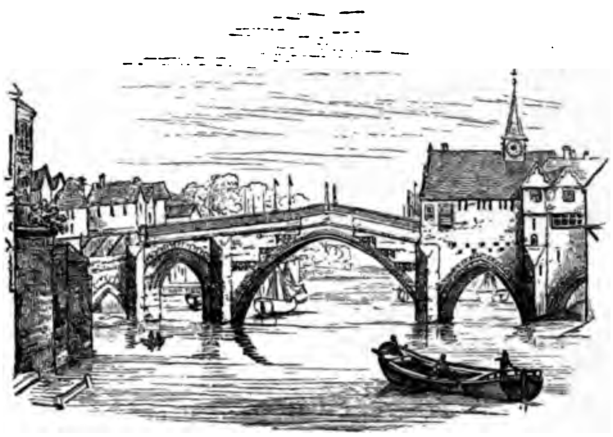


FIG. 115.—Old Bridge at York.

Heck mentions a bridge at Loyang in China, said to have a length of 26,800 feet, and another at Focheu 22,000 feet long, both from 60 to 70 feet wide.

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§ 76. 1700 to 1817.—Old Westminster Bridge (Labelye) and Old Blackfriars Bridge (Mylne), both of which have now been removed, were built in the middle of the 18th century. Their failure after so short a period was due to a defective system of foundation and to the increased scour caused by the removal of old London Bridge.

ont-y-tu
rydd.

The Pont-y-tu-Prydd over the River Taff near Newbridge is shown in fig. 117. The arch measures 140 feet between the abutments, and has a rise or versed sine of 35 feet. The width of the soffit is 15 feet 10 inches at the springing, diminishing to 14 feet 5 inches at the crown by six offsets on each face. The roadway is 11 feet wide over the centre of the arch. The arch stones on the face are 2 feet 6 inches deep, the rest of the ring being rubble masonry. This bridge was built by William Edwards, a self-educated country mason. It was completed in 1750 after the failure of a similar structure, in which the weight of the haunches was excessive and forced up the crown, the depth of which was very small. This failure led to the adoption of the pierced spandrils.

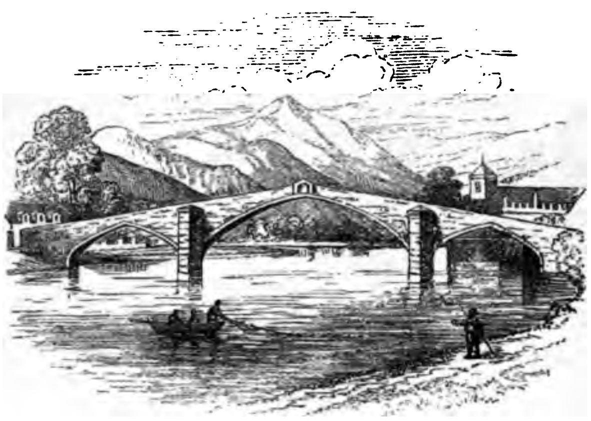


FIG. 116.—Llanrwst Bridge, Wales.

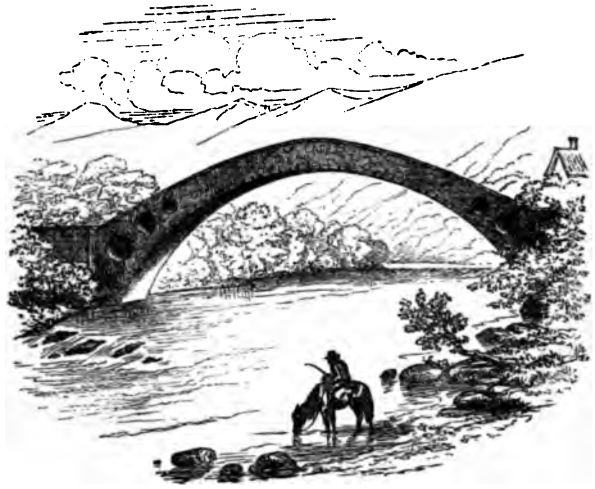


FIG. 117.—Pont-y-tu-Prydd.

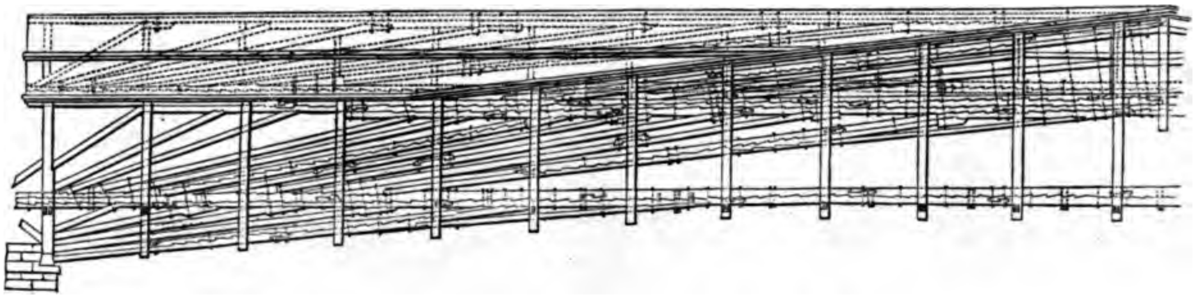


FIG. 118.—Half Truss of Wittingen Bridge.

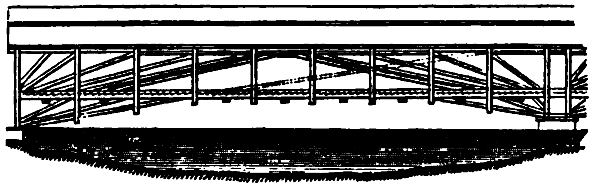


FIG. 119.—First Arch of Schaffhausen Bridge.

Fig. 118 shows half of the truss for the bridge of Wittingen, built in 1758 by the brothers Grubenmann, probably the finest specimen of a wooden bridge that has ever been constructed; the design might be analyzed as consisting of a series of superposed trusses, as in fig. 87, which represents the bridge at Schaffhausen built by the same engineers or village carpenters. The Schaffhausen bridge (fig. 119), destroyed by French troops in 1799, had two openings, one of 172 feet and the other 193 feet. The Wittingen bridge, burnt shortly afterwards, had a span of 390 feet, being the largest opening ever spanned by wood.

Mr Smiles states that the first attempt to build a cast-iron bridge was made in 1755 at Lyons, and that one of the arches was put together in a builder's yard, but that the project was abandoned as too costly. **Coalbrookdale.** Mr Abraham Darby, the owner of iron-works of Coalbrookdale, was the first person who actually erected a cast-iron arch. This bridge (fig. 120) crosses the Severn by a span of 100 feet, near the town of Ironbridge, which has sprung up in the neighbourhood. Each of the ribs consisted of two pieces. The design is a bold and original one, and has been practically successful. Wearmouth Bridge, completed in 1796, is an arch built of open cast-iron panels, acting as voussoirs; the span is 236 feet, with a rise of 34 feet; the springings begin 95 feet above the bed of the river; and the width of the bridge was 32 feet. It contained 214 tons of cast-iron and 46 tons of wrought iron. The name of Thomas Paine, the well-known author, has been associated with the design of this bridge; but Mr L. D. B. Gordon (first Professor of Engineering in Glasgow) assures the writer that after careful investigation he finds that Rowland Burdon, member for the county, was engineer, architect, and paymaster for this remarkable bridge. It was repaired and widened by Robert Stephenson in 1858. The bridge erected by Telford at Craigellachie (fig. 121), over the Spey, in the beginning of this century, shows a great advance in the conception of what was the safest form in which to apply cast-iron to an arch.

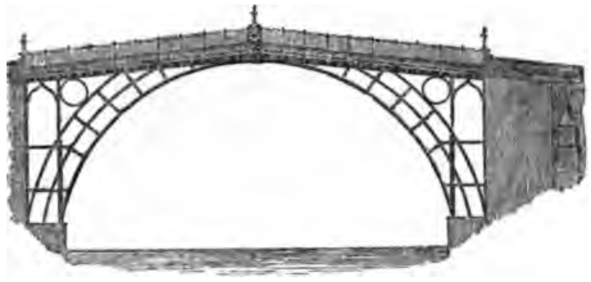


Fig. 120.—Coalbrookdale Bridge.

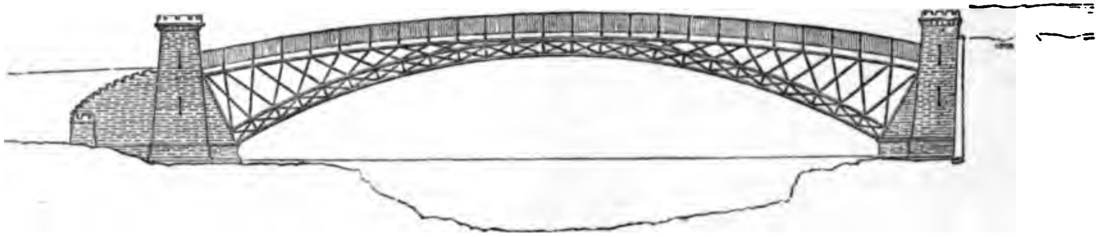


Fig. 121.—Cast-Iron Bridge at Craigellachie

§ 77. 1817 to 1845.—*London Bridge and Waterloo Bridge.*—London new bridge (fig. 40, *supra*, and Plate XVIII. fig. 2), is as fine an example of the modern stone arch bridge as can be found. The shape of the arches, the variation in their span, the slight curvature of the roadway, and the simple yet bold architectural details, are combined so as to produce a singularly beautiful structure. It is now insufficiently wide for the traffic it has to convey, but all who value beauty must earnestly desire that it may not be disfigured by having overhanging footpaths fitted to it as has been frequently proposed. London can well afford to pay for new bridges, but can by no means afford to part with a single object of real beauty.

The design was made by Mr George Rennie, and the acting engineer was his brother, Sir John Rennie. The centre arch has a span of 152 feet, and rises 29 feet 6 inches above Trinity high water mark; the arches on each side of the centre have a span of 140 feet, and the abutment arches 130 feet. The total length of the bridge is 1005 feet, its width from outside to outside 56 feet, and height above low water 60 feet. The two centre piers are 24 feet thick, the exterior stones are granite, the interior, half Bramley Fall and half from Painshaw, Derbyshire.

The voussoirs of the centre arch (all of granite) are 4 feet 9 inches deep at the crown, and increase to not less than 9 feet at the springing. The general depth at which the foundations are laid is about 29 feet 6 inches below low water. Seven years and a quarter were spent in the construction of London bridge, which was opened in 1831. The total cost was £1,458,311, but the contractor's tender for the bridge alone was £425,081.

Waterloo Bridge, Plate XVIII. fig. 3, is another fine structure of the same character (1817).

Introduction of Suspension Bridges.—It will be observed that from the earliest ages in which we have records of the construction of permanent bridges until very lately, the stone or brick arch has been the structure principally relied on. Timber bridges more or less permanent have also been employed for great spans, as in the noble bridges erected by the brothers Grubenmann (1757); and after the construction of the bridge at Coalbrookdale (1777) cast-iron was not unfrequently employed in England. The theory of the metal arch was, however, very imperfectly understood, and the great metal arch of **Southwark Bridge.** Southwark bridge (completed 1819), Plate XVIII. fig. 4 (largest span 240 feet), is little more than a heavy and wasteful imitation of a stone ring. By the use of timber or cast-iron instead of stone, the opening which a bridge could span was, however, somewhat increased. An immense stride in this direction was made when suspension bridges were introduced. **Tees Bridge.** A bridge of this kind over the Tees, 70 feet in length, was built in 1741 for the use of miners. Similar bridges are also said to have been used by Mr Finley in America, but the introduction of the modern suspension bridge practically dates from about 1820. **Galashiels.** (Galashiels bridge, 112 feet in length, was constructed in 1816, **Peebles.** also a bridge of similar dimensions at Peebles over the Tweed). **Menai Bridge.** In 1819 Telford began the construction of the Menai suspension bridge (Plate XIX. fig. 2), in which the span of the catenary is 570 feet and the dip 43 feet. The success of this structure led to the construction of many other large suspension bridges, as at Fribourg (span 870 feet), Hammersmith (span 422 feet), Pesth (span 666 feet). This form of bridge was not, however, found suitable for railway traffic; and on the introduction of railways engineers were for many years dependent on stone, brick, or cast-iron arches.

Britannia Bridge.

§ 78. *Britannia Bridge*, 1845.—The design by Robert Stephenson of a bridge to carry the Chester and Holyhead Railway across the Menai Straits led to a complete revolution in engineering practice. Mr Stephenson's first conception was that of a tube partly carried by chains. This would have practically been a suspension bridge stiffened by a girder. Under Mr Stephenson's directions, experiments and calculations on the strength and best form of tubes were made by Mr William Fairbairn (Sir William Fairbairn) and Mr Eaton Hodgkinson. In the course of the experiments it was found that the tube could be made self-supporting over the desired span of 460 feet; and in consequence of this discovery the Conway and Menai tubular bridges were built, being the first great examples of properly designed girders. Some disputes arose as to the real inventor of these bridges. Sir William Fairbairn justly claimed the great merit of first perceiving that the girder might be self-supporting. Mr Hodgkinson had, perhaps, the smallest part in the design, but the shares of Fairbairn and Stephenson respectively cannot be very rigorously apportioned; nor is this now of much consequence. Both engineers were men of extraordinary merit, and co-operated in producing the great revolution in practice which has led to the adoption of the wrought iron girder as the most common type of bridge. The first train passed through the Britannia Bridge in 1850. The following description of the structure appeared in the 8th edition of the present work. It will be seen that this description is to some extent a defence of the design against criticisms asserting that the structure was unnecessarily heavy and costly. It is true that a considerably lighter bridge could now be built, but some prudence in introducing so great a novelty was certainly commendable.

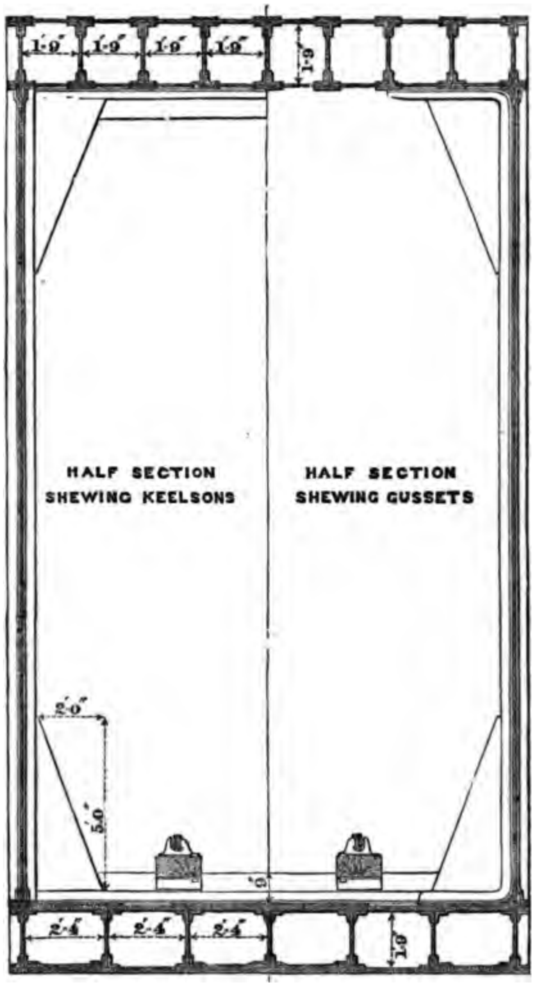


FIG. 122.—Britannia Bridge (Cross Section of Tubular Girder).

"The Britannia Bridge which carries the Chester and Holyhead Railway over the Menai Straits (figs. 122 and 123, and Plate XIX. fig. 1) consists of two independent continuous wrought iron tubular beams, 1511 feet in length, and weighing 4680 tons each, independent of the cast-iron frames inserted at their bearings on the towers. They are 15 feet wide, and vary in depth from 23 feet at the ends to 30 feet at the centre. They rest on two abutments and three towers of masonry at a height of 100 feet above high water. The roadway is laid along the bottom, viz., one line of rails in each tube. The centre or Britannia tower, which is altogether 230 feet high, is built on a rock in the middle of the Straits. The bridge has thus four spans, viz., two spans of 460 feet over the water, and two spans of 230 feet over the land. On each side the weight of a single span of 470 feet is 1587 tons, and of a span of 242 feet 630 tons. These tubes repose solidly on the centre tower, but repose on roller beds on the land towers and abutments. Now, these gigantic dimensions are by no means the only remarkable features in this work. The opponents of the Holyhead Road had imposed conditions on the Chester and Holyhead Railway which were thought insurmountable with respect to this bridge. The navigation was not to be interrupted—no scaffolding could thus be used—and the clear height of 100 feet was to be retained throughout,—arches being objected to unless the springing and not the centre was at this elevation. The tides set through this portion of the Strait with a velocity of 9 miles per hour, and the quiet water at each turn of the tide lasts but for a very short space of time. The tubes were designed to meet all these requirements; they were so constructed at a considerable distance from their permanent site on the shores of the Straits; they were floated upon pontoons upon these rapid tides to the base of the towers; and they were then drawn up by hydraulic presses to their required height. They were here united through the towers by the insertion of shorter lengths, and ultimately brought into the condition of continuous beams as regards strain, by the means employed for their junction. It is evident such structures would be designed specially for such varied circumstances, for example :—

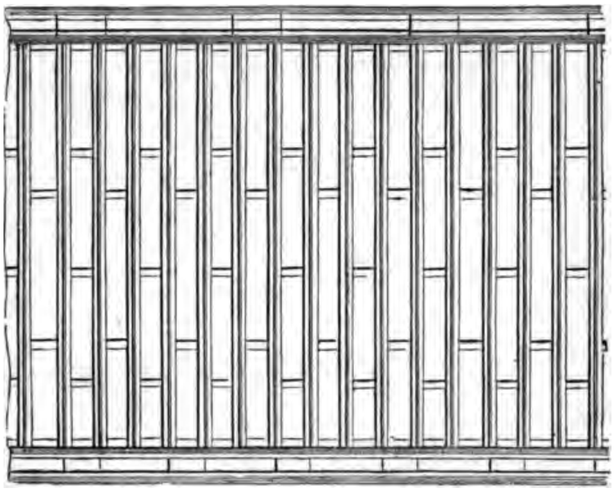


FIG. 123.—Britannia Bridge (Part Elevation of Tubular Girder).

"As soon as they were completed on temporary platforms, these platforms were removed, and they became isolated beams; the ends were accordingly strengthened with cast and wrought iron framing for this special object, and had they always remained there the sides might have been throughout considerably lighter than they are; they now weigh nearly 40 per cent. of the whole weight. But in the next operation, that of floating, the tubes were liable to be supported at *any point* of their length, besides being subjected to chances of considerable distortion, and to disasters which on more than one occasion did actually threaten their entire destruction. The stiffening frames and gussets, which in an ordinary girder would have only been necessary at the ends, became therefore necessary throughout the whole length, and even the top and bottom were considerably modified, as it is evident that while overhanging the pontoons on each end to the extent of 70 feet, the top, instead of being in compression was thrown into extension; the weight of the tubes was consequently much increased by these arrangements. Again, they had to be raised by being suspended freely from four chains. Provision for this suspension from such limited attachment had also to be made of a totally opposite character from that made for their vertical support when on their bed; and, ultimately, when raised to their place, they remained no longer independent beams, but were converted into continuous beams, parts before in tension being now thrown into compression, and *vice versa;* while the ends which were before subject to no horizontal strain were now exposed to greater strain than even the centre of the span. And, last of all, during the act of raising one of these enormous masses, the press from which it was suspended burst, and one end of the beam fell through a space of no less than 9 inches on to a loose uneven heap of planks beneath it, bulging in the bottom plates, breaking all the castings, distorting seriously the sides and stiffening frames; while the broken press itself, which descended from a height of about 100 feet above, broke through the top plates and completed the crippling of the whole section of support. It may surely be doubted whether anything but a tube could have stood such unexampled violence; and in proportioning the parts of a structure destined for such usage, the mere consideration of the strain to which as an ordinary beam it would be subjected, formed but a part of the problem; no direct comparison can therefore be made between the weight of this bridge and an ordinary beam. If this were the case with the large spans, it is still more so with the small spans of 230 feet, which as simple beams would weigh only 230 tons each, whereas their actual weight is 650 tons. But it must be borne in mind that as regards the bridge itself these small spans were not required at all, and that they were merely designed and used as counterpoises for the large tubes, for the important purpose of converting them into continuous beams by their overhanging weight. By examining their detail, it will be found they are designed solely for this special purpose, their use as beams being made entirely subsidiary.

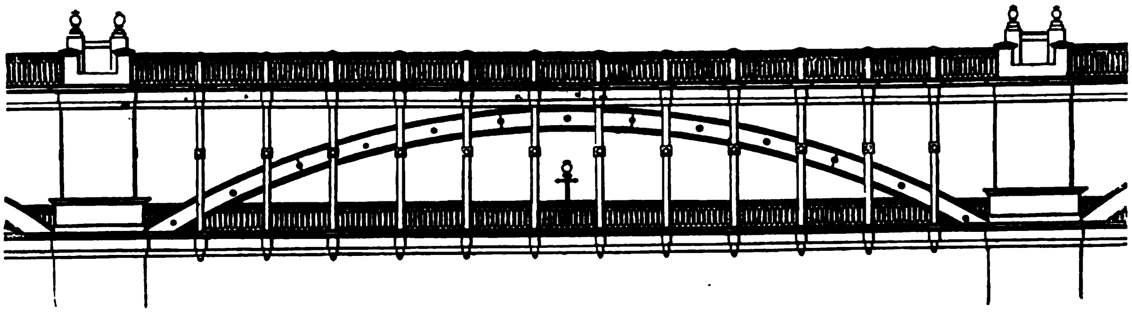


FIG. 124.—Elevation of Bowstring Arch, High-Level Bridge, Newcastle

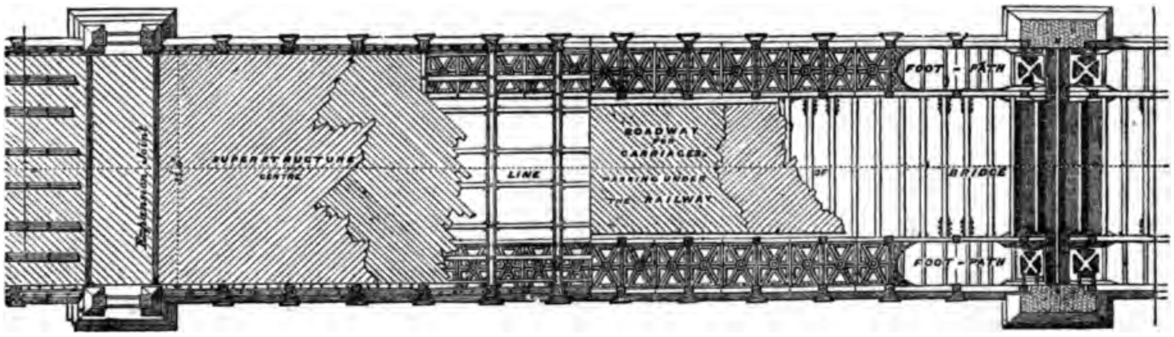


FIG. 125.—Plan of High-Level Bridge, Newcastle.

"Some misapprehension exists on the object and importance of the cells of which the top and bottom of these tubes is composed. These cells are rectangular, there being eight of them in the top and six of them in the bottom, and they run throughout the bridge. With respect to their importance, it must be observed that the whole section of the top of the Britannia tube at the centre is 648·25 square inches, and of the bottom 585·43 square inches, and that the tube is 15 feet wide; the thickness of a single plate to ensure this section would therefore have been 2·7 inches for the top, and 2·3 inches for the bottom; and had such a plate been procurable, nothing better could have been desired, and the cells would be unnecessary. Such a thing, however, is evidently impossible, and the engineer in this, as in numberless other details, had to adopt what he could obtain; now the arrangement of the plates in cells is almost the only conceivable arrangement possible for obtaining the required section, allowing access, at the same time, to every part for construction and future maintenance. This alone led to their use in the bottom of the tube, where their form was totally unimportant. With respect to the top, however, it was of great importance, since thick plates could not be had, to ascertain the best form of cell for resistance to compression that could be devised with thin plates. A series of valuable experiments by Mr Eaton Hodgkinson led to the use of the rectangular cells as actually used, not because such form presented any peculiar advantage over any other form, as some have imagined, but because these experiments demonstrated that cells of that magnitude and thickness were independent of form, and are crushed only by the actual crushing of the iron itself; under these circumstances, the square cells were used as the best practical method of obtaining the sectional area required.

"Similar misapprehension also exists as to the considerations which led to the rectangular form of the tubes themselves. Now, the result of direct experiments made with round, oval, and rectangular tubes—there being precisely the same section and weight in all three, and, consequently, different depths—was, that the circular tube was the weakest, and the oval tube the strongest, the rectangular form being intermediate. The oval tube was, indeed, first studied with a view to its use. Its form, however, was not favourable,—neither for its actual construction, nor for its connection with the suspension chains which were originally intended to be used in the erection; and practical considerations, in this case, also compelled the use of the rectangular tube. It must also be remarked, that the result of experiments made on round, oval, and rectangular wrought iron tubes, when reduced to the same depth and compared, was in favour of the rectangular form, although within ordinary limits the form was not proved to be a matter of very great importance. It may be added, that this bridge has now been in use six years, that the deflection has been carefully tested, from time to time, with the utmost precision, and that not the slightest perceptible increase has taken place during that period. The care with which the painting has been attended, and the protection afforded by the roof, have also entirely preserved it from the slightest damage by oxidation; and it is difficult to conceive that even the lapse of centuries can in any way affect such a structure, or to doubt that it will remain one of the most durable, as it certainly is one of the most remarkable monuments of the enterprise of the present century."

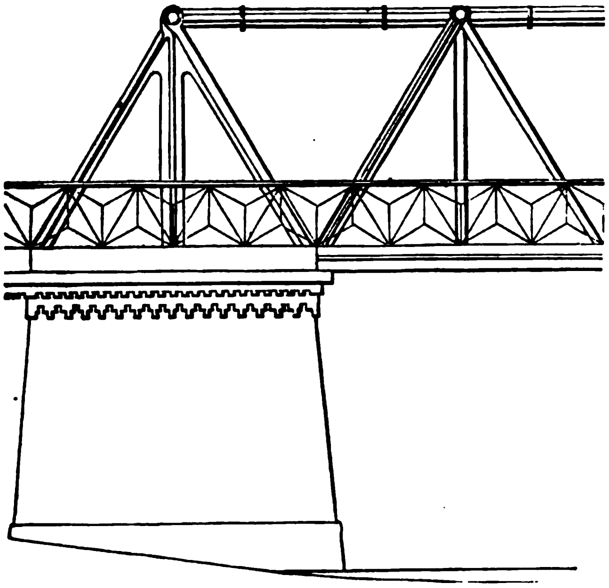


Fig. 126.—Newark Dyke Bridge.

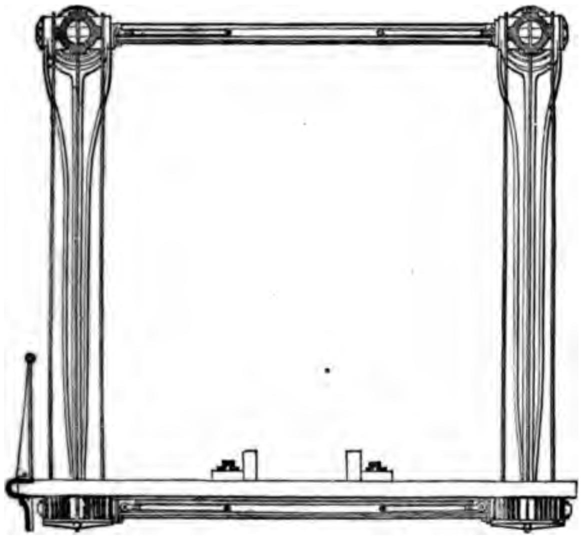


Fig. 127.—Section of Newark Dyke Bridge.

Newcas High-L Bridge.

§ 79. *Newcastle High-Level Bridge. Newark Dyke Bridge. Crumlin Viaduct.*—The High-Level Bridge at Newcastle (figs. 124 and 125, also Plate XIX. fig. 3) is a fine example of the true bowstring arch, in which there is no cross bracing. This bridge is also described at great length in the 8th edition; but the type cannot be recommended for imitation, being essentially more expensive and heavier than a true girder. The bridge was opened by the Queen in 1849. The design was therefore made almost at the same time as that for the Britannia Bridge, and is chiefly interesting as showing a transitional form intermediate between the arch and beam. The bridge has six spans, each of 125 feet, and the superstructure is supported on stone piers and abutments, the height to the soffit above high water being 83 feet. The arched ribs are cast-iron, and the ties wrought iron. 4728 tons of cast-iron and 321 tons of wrought iron were employed in the superstructure. There are two roadways, the carriage roadway passing under the railway. The bridge cost £243,000.

The solid or continuous plate girder soon led to the introduction of open frames, designed on similar principles.

Newark Dyke Bridge.

Newark Dyke Bridge (the earliest example of a Warren girder bridge) carries the Great Northern line over a branch of the Trent near Newark. It was erected (1851–53) under the direction of Mr Joseph Cubitt from the designs of Mr Charles Wild.

This bridge (figs. 126 and 127) consists of four independent girders, viz., two for each line of railway. The roadway is beneath the girder. The top flange of each girder consists of a series of cast-iron pipes butting end to end; the lower flange consists of wrought iron links, and the flanges are connected by diagonals forming a series of equilateral triangles, and these diagonals are alternately struts and ties. The ties are formed of wrought iron; and the struts of cast-iron: the length of each side of these triangles is 18 feet 6 inches.

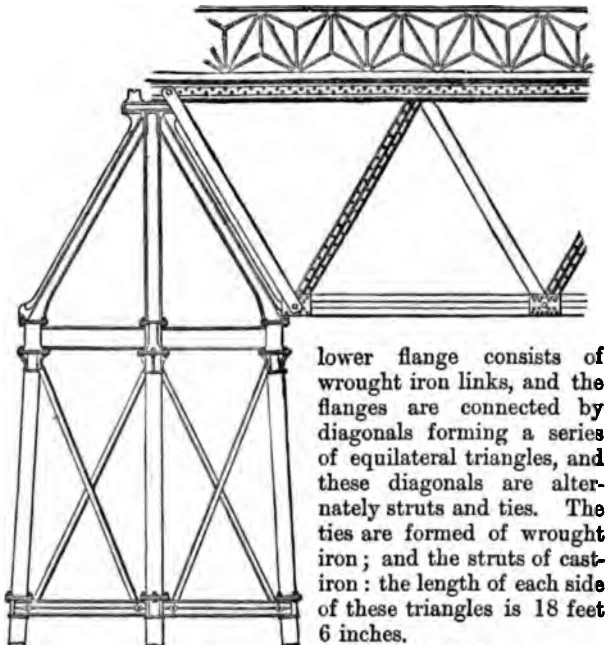


Fig. 128.—Part of Crumlin Viaduct.

The length between the supports is 259 feet, and the depth from centre to centre of the joint pins is 16 feet. The clear span between the abutments is 240 feet 6 inches.

The weight of iron is 244 tons 10 cwt., of which 106 tons 5 cwt. is wrought iron, and 138 tons 5 cwt. cast-iron, to which must be added 50 tons for the platform, making the total weight of each bridge 294 tons 10 cwt. The cost, exclusive of the masonry of the abutments, and of the permanent rails, but inclusive of the staging for fixing and the expense of testing, was £11,003.

Crumlin Viaduct.

The Crumlin Viaduct, begun in 1853, and completed in 1857 (fig 4, Plate XIX.), is a fine example of the Warren girder; it was erected on the Taff Vale Extension Railway under Messrs Liddell and Gordon as engineers, by Mr T. W. Kennard as contractor. The following description is compiled from that given in Mr Humber's work:—The length of the bridge is 1800 feet, divided into two parts, one consisting of seven spans of 100 feet, and the other of three similar spans. The greatest height of the road-way above the surface of the water is not less than 200 feet. The piers are formed of cast-iron hollow columns, each 17 feet long and 1 foot in diameter; the thickness of the metal varies from 1 inch to ⅞th inch; these columns are arranged in tiers, each containing fourteen columns, the distance between which at the base of the pier measures 13 feet 6 inches, excepting between the centre rows, where it measures 6 feet throughout the height. The wrought iron girders are 150 feet in length and 14 feet 6 inches deep. Some details of the construction are shown in fig. 128.

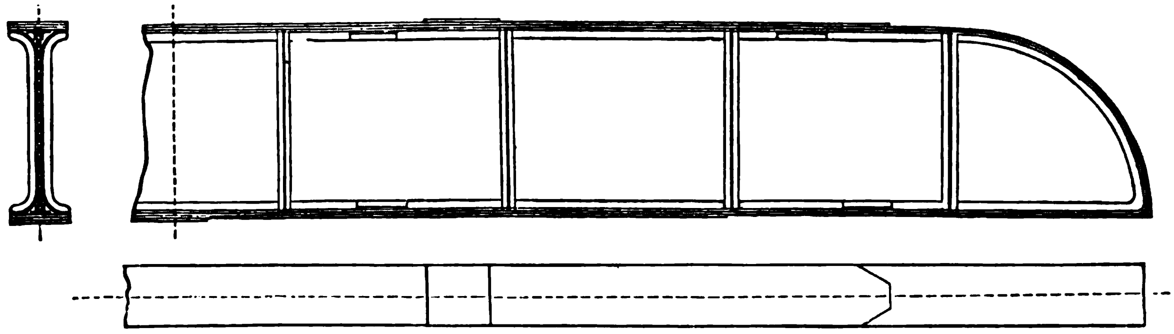


FIG. 129.—Common type of Wrought Iron Girder.

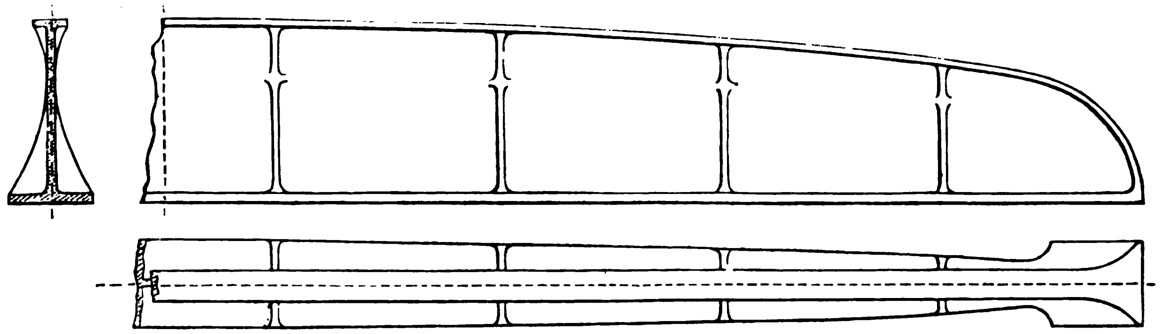


FIG. 130.—Common type of Cast-Iron Girder.

Fig. 129 shows a plan and elevation and cross section for half a wrought iron girder of a usual type for small spans. Covering plates are used to connect the main plates of the top and bottom webs, and stiffening angle irons are shown at the sides.

Similarly, fig. 130 shows a plan and elevation of a cast-iron girder of a usual type.

§ 80. *Niagara Suspension Bridge.*—Fig. 5, Plate XIX., shows Niagara Suspension Bridge, a structure described as follows in the 8th edition:—

"It crosses the Niagara River at a height of 245 feet above the water by a single span of 821 feet 4 inches, and forms the connecting link between the American States and Canada.

"The superstructure may be best described as a hollow rectangular box, 18 feet deep and 24 feet wide, on the top of which the railway is laid, while the bottom, which is 25 feet wide, forms the roadway for public traffic—both these floors are constructed of timber beams; and each connecting side consists of a row of double posts or uprights of timber, each pair being 5 feet apart; between them wrought iron diagonal bars are made to pass, extending each way to the fourth pair of posts at an angle of 45 degrees. The upper or railway floor is suspended from two wire cables at intervals of 5 feet, and the lower floor is suspended at similar intervals from two other wire cables which have a deflection of 10 feet more than the upper ones; these cables, four in number, are each 10 inches in diameter, and composed of seven strands, each containing 520 wires, making a total of 3640 wires. One strand forms the axis round which the other six are twisted. Sixty wires are equal to 1 square inch of solid section; therefore the total area of each cable is 60·4 square inches, or the total sectional area of iron supporting the structure is 241·6 square inches.

"Each cable rests upon a separate saddle, there being two on the top of each of the four towers. The saddles are placed on ten cast-iron rollers, 5 inches diameter and 25¼ inches long, which bear upon cast-iron plates 8 feet square and 2½ inches thick, strengthened by three parallel flanges which form two compartments for the reception of the saddles.

"The ends of the cables are attached to cast-iron shoes, in each of which is inserted a wrought iron pin which forms the connection with the anchor chains. These anchor chains are each embedded in a solid shaft of masonry 7 feet by 3 feet, enlarged at the bottom to form a chamber 8 feet square cut in the rock. The shafts are sunk to a depth of 25 feet on the New York side, and 35 feet on the Canada side.

"Each anchor chain is composed of nine links, the eight lower links being 7 feet long, and the ninth or uppermost 10 feet long. The lowest link consists of seven wrought iron bars, 7 inches by 1·4 inches each, and amounting collectively to an area of 69 square inches. They are secured to a cast-iron anchor plate, by a pin 3½ inches diameter. From the fourth link the chain curves, and the section is gradually increased to an area of 93 square inches. There are two towers at each end of the bridge, based upon a mass of masonry 60 feet by 20 feet, which is pierced by an arch 19 feet wide, forming the entrance to the lower roadway. The towers are 60 feet high, 15 feet square at the base, and 8 feet square at the top.

"Above the floors are 64 diagonal stays, extending from the saddles to the suspenders, amongst which they are equally distributed; they are formed of wire-rope 1⅜th inches diameter. There are also 56 stays attached at their upper extremities to the soffit of the bridge, and at their other ends well anchored to the rocks below. The superstructure is thus tied down as well as suspended, and all undulations directly resisted.

"The bridge was commenced in September 1852, and opened for traffic in March 1855. The total cost was £80,000."

The use of two chains of different versed sines is certainly a defect in this design.

There are several other suspension bridges in the United States of great span, *e.g.*, Cincinnati, 1057 feet; Brooklyn 1600 feet.

§ 81. *Saltash, Victoria, and Coblentz Bridges. Fink Truss.* —Fig. 131 shows one span of Saltash Bridge erected by Brunel. The span is 455 feet. The pier is a column or circular pillar of solid masonry, 35 feet diameter and 96 feet high from the rock foundation to above high-water mark. Upon this are placed four octagonal columns of cast-iron, 10 feet diameter, carried up to the level of the roadway, which is 100 feet above high-water mark. **Saltash.**

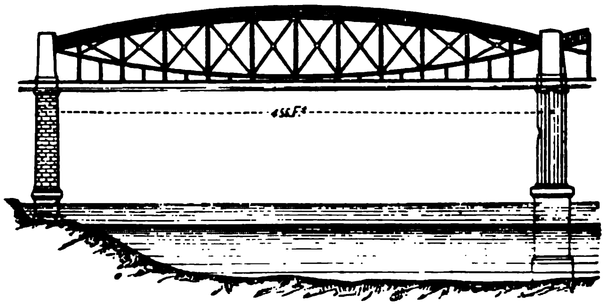


Fig. 131.—Span of Saltash Bridge.

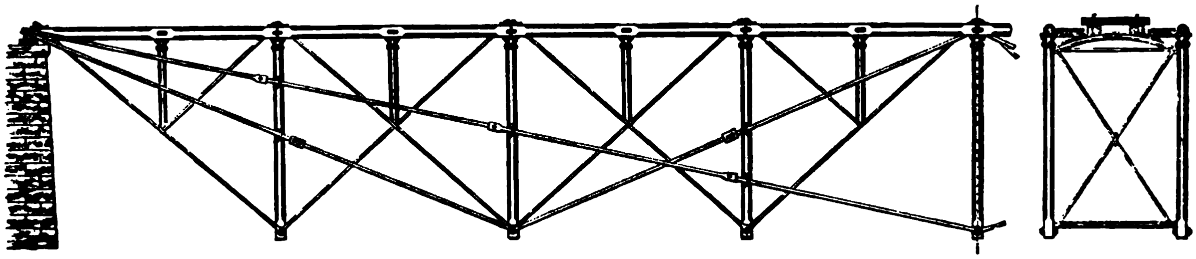


Fig. 132.—Fink Truss.

Victoria Bridge, Montreal. The Victoria Bridge over the St Lawrence at Montreal is a tubular bridge of great length (7000 feet), chiefly remarkable for its ice breakers, shown in fig. 93.

Fig. 132 shows some details of a Fink truss as used in America. The mode of computing the stresses on this truss has already been explained in § 59. All the struts are cast-iron tubes.

Fig. 133 shows one of the wrought iron arches of a bridge over the Rhine at Coblentz. The bridge consists of three spans of about 315 feet each. **Coblentz.**

§ 82. *St Louis and Illinois Bridge.*—The St Louis and Illinois bridge over the Mississippi (fig. 5, Plate XVIII.) is the finest example of a metal arch yet erected. It is described as follows by Sir Charles A. Hartley who visited it in 1873:— **St Louis Bridge.**

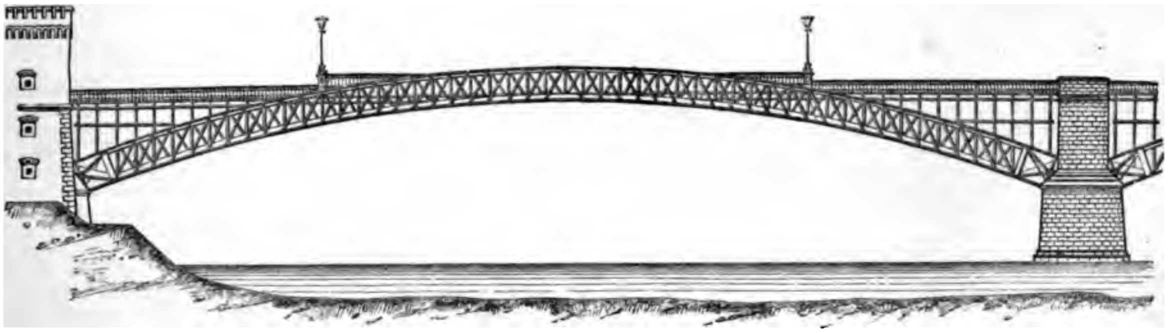


Fig. 133.—Arch of Bridge at Coblentz.

"The Mississippi at St Louis is confined to a single channel 1600 feet wide and 8 feet deep at extreme low water by an embankment or levee on the Illinois side, which is carried up to the level of extreme high water, at which time the width is augmented to 2200 feet. Both shores are revetted below the low water, some with rubble stones, and protected by the wharf pavements above that line. The extreme range between high and low water is 41 feet. Owing to the narrow gorge through which the whole volume of the Missisippi flows the variations in the bed of the river are very great. Captain James B. Eads, M. Inst. C.E., the distinguished engineer who designed the bridge and superintended its construction, informed the author that a rise of 13 feet less than high-water mark caused a scour of 18 feet, and that in the freshet of 1870 the scour reached a depth of 51 feet below low-water mark alongside the east pier. These facts induced him to believe it possible that the scour, at times of extraordinary high flood, might extend even to the rock itself. He therefore determined to establish the piers and abutments on the rock; and this was done by means of caissons provided with air chambers and locks at depths for the east pier and east abutment reaching 136 feet below high-water mark, or 110 feet from the surface of the water where the foundation work was actually performed. This feat, which was satisfactorily executed in 1870–71, is quite unprecedented in the annals of engineering.

The piers and abutments are composed of coursed rubble masonry up to low-water mark. Above this level they are faced

Fig. 1.

LONDON OLD BRIDGE.

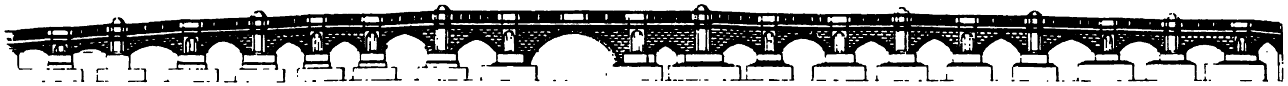


Fig. 2.

LONDON NEW BRIDGE.

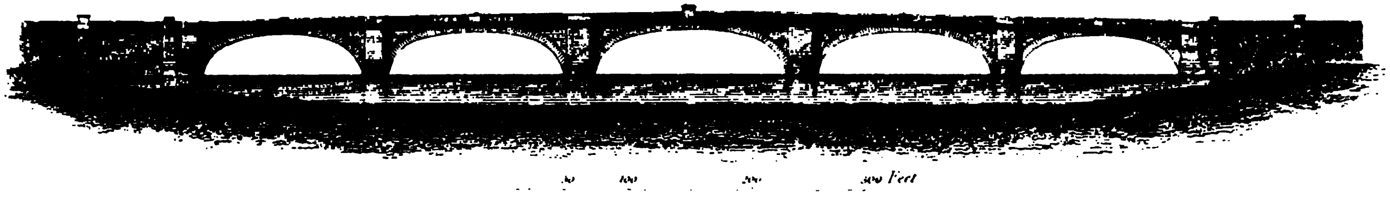


Fig. 3.

WATERLOO BRIDGE.

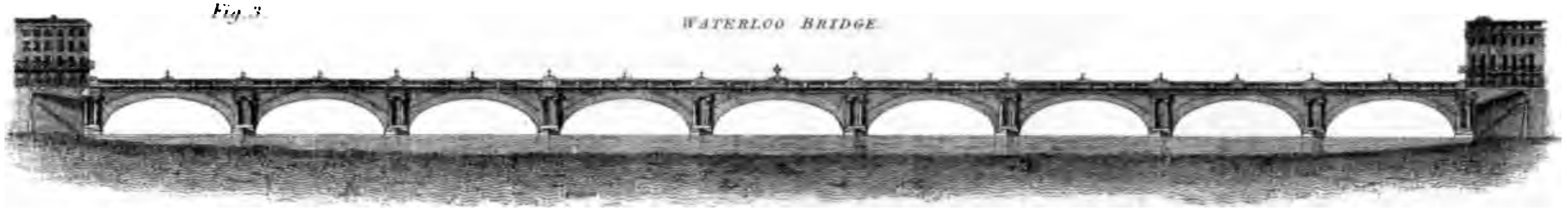


Fig. 4.

SOUTHWARK BRIDGE

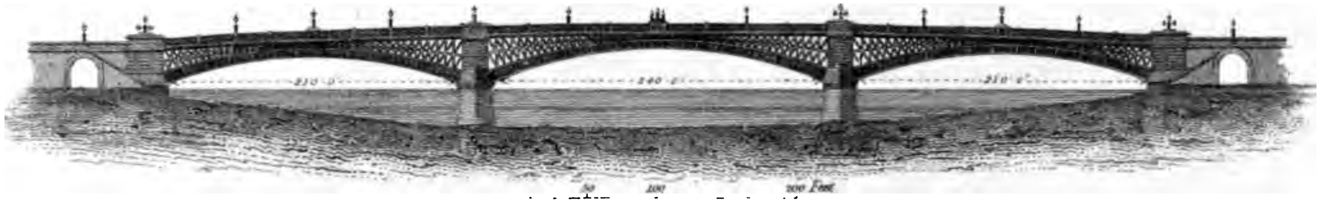
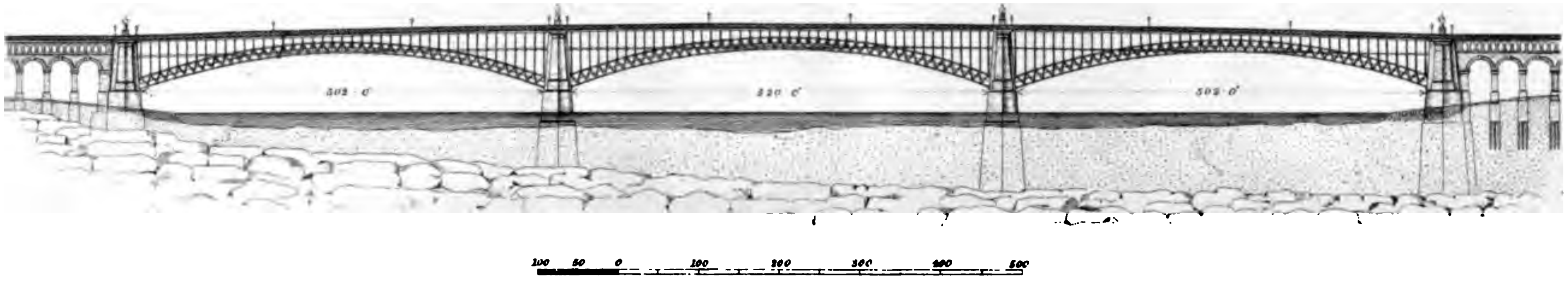


Fig. 5.

ILLINOIS & St LOUIS BRIDGE.



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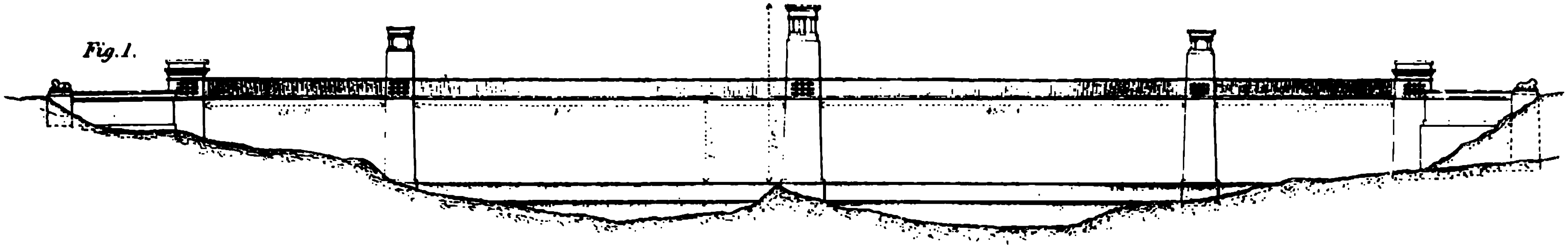
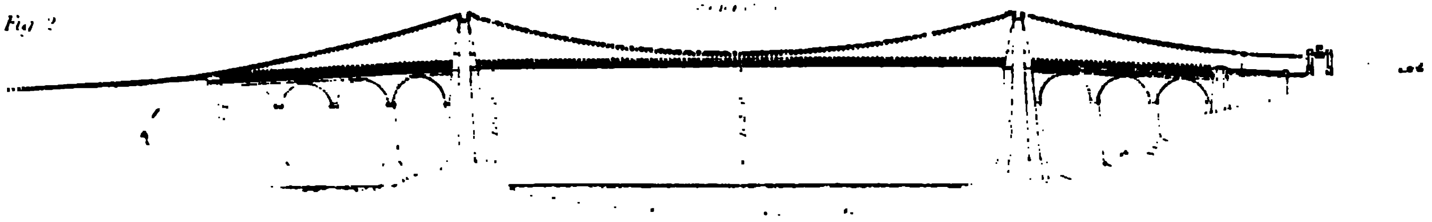
BRITANNIA TUBULAR BRIDGE.

Fig. 1.



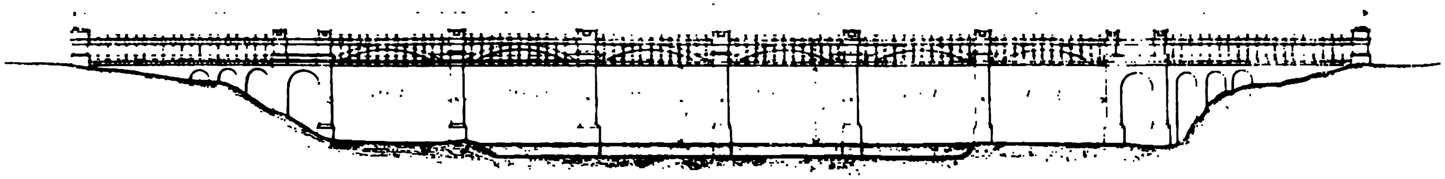
MENAI SUSPENSION BRIDGE

Fig. 2



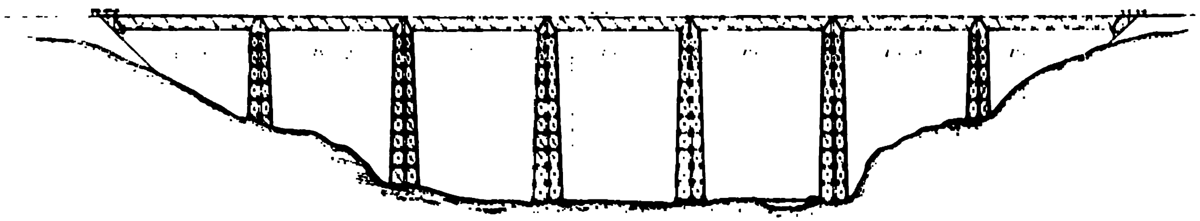
HIGH LEVEL BRIDGE NEWCASTLE.

Fig. 3



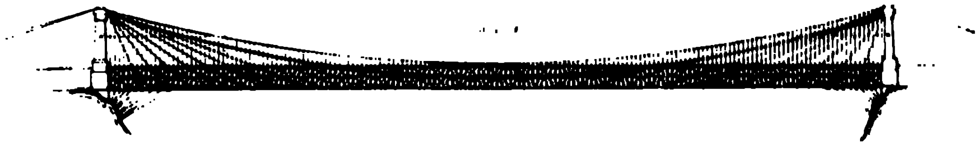
CRUMLIN VIADUCT.
South Wales.

Fig. 4



NIAGARA SUSPENSION BRIDGE.

Fig. 5



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